Package 'evd'

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Title Functions for Extreme Value Distributions

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Imports stats, grDevices, graphics

Suggests interp

Description Extends simulation, distribution, quantile and density functions to univariate and multivariate parametric extreme value distributions, and provides fitting functions which calculate maximum likelihood estimates for univariate and bivariate maxima models, and for univariate and bivariate threshold models.

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abvevd *Parametric Dependence Functions of Bivariate Extreme Value Models*

Description

Calculate or plot the dependence function A for nine parametric bivariate extreme value models.

Usage

```
abvevd(x = 0.5, dep, asy = c(1,1), alpha, beta, model = c("log", "alog",
   "hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
     rev = FALSE, plot = FALSE, add = FALSE, lty = 1, lwd = 1, col = 1,
    blty = 3, blwd = 1, xlim = c(0,1), ylim = c(0.5,1), xlab = "t",
    ylab = "A(t)", ..., )
```


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Details

Any bivariate extreme value distribution can be written as

$$
G(z_1, z_2) = \exp\left[-(y_1 + y_2)A\left(\frac{y_1}{y_1 + y_2}\right) \right]
$$

for some function $A(\cdot)$ defined on [0, 1], where

$$
y_i = \{1 + s_i(z_i - a_i)/b_i\}^{-1/s_i}
$$

for $1 + s_i(z_i - a_i)/b_i > 0$ and $i = 1, 2$, with the (generalized extreme value) marginal parameters given by (a_i, b_i, s_i) , $b_i > 0$. If $s_i = 0$ then y_i is defined by continuity.

 $A(\cdot)$ is called (by some authors) the dependence function. It follows that $A(0) = A(1) = 1$, and that $A(\cdot)$ is a convex function with $\max(x, 1-x) \leq A(x) \leq 1$ for all $0 \leq x \leq 1$. The lower and upper limits of A are obtained under complete dependence and independence respectively. $A(\cdot)$ does not depend on the marginal parameters.

Some authors take $B(x) = A(1-x)$ as the dependence function. If the argument rev = TRUE, then $B(x)$ is plotted/evaluated.

Value

abvevd calculates or plots the dependence function for one of nine parametric bivariate extreme value models, at specified parameter values.

See Also

[abvnonpar](#page-4-1), [fbvevd](#page-30-1), [rbvevd](#page-12-1), [amvevd](#page-6-1)

Examples

```
abvevd(dep = 2.7, model = "hr")abvevd(seq(0,1,0.25), dep = 0.3,asy = c(.7,.9), model = "alog")abvevd(alpha = 0.3, beta = 1.2, model = "negbi", plot = TRUE)bvdata <- rbvevd(100, dep = 0.7, model = "log")
M1 <- fitted(fbvevd(bvdata, model = "log"))
abvevd(dep = M1["dep"], model = "log", plot = TRUE)
abronpar(data = bvdata, add = TRUE, lty = 2)
```
abvnonpar *Non-parametric Estimates for Dependence Functions of the Bivariate Extreme Value Distribution*

Description

Calculate or plot non-parametric estimates for the dependence function A of the bivariate extreme value distribution.

Usage

```
abronopar(x = 0.5, data, epmar = FALSE, nsloc1 = NULL,nsloc2 = NULL, method = c("cfg", "pickands", "tdo", "pot"),
    k = nrow(data)/4, convex = FALSE, rev = FALSE, madj = 0,
   kmar = NULL, plot = FALSE, add = FALSE, lty = 1, lwd = 1,
   col = 1, blty = 3, blwd = 1, xlim = c(0, 1), ylim = c(0.5, 1),
   xlab = "t", ylab = "A(t)", ...)
```


Details

The dependence function $A(\cdot)$ of the bivariate extreme value distribution is defined in [abvevd](#page-2-1). Nonparametric estimates are constructed as follows. Suppose (z_{i1}, z_{i2}) for $i = 1, \ldots, n$ are n bivariate observations that are passed using the data argument. If epmar is FALSE (the default), then the marginal parameters of the GEV margins are estimated (under the assumption of independence) and the data is transformed using

$$
y_{i1} = \{1 + \hat{s}_1(z_{i1} - \hat{a}_1)/\hat{b}_1\}_+^{-1/\hat{s}_1}
$$

and

$$
y_{i2} = \{1 + \hat{s}_2(z_{i2} - \hat{a}_2)/\hat{b}_2\}^{-1/\hat{s}_2}_+
$$

for $i = 1, \ldots, n$, where $(\hat{a}_1, \hat{b}_1, \hat{s}_1)$ and $(\hat{a}_2, \hat{b}_2, \hat{s}_2)$ are the maximum likelihood estimates for the location, scale and shape parameters on the first and second margins. If nsloc1 or nsloc2 are given, the location parameters may depend on i (see [fgev](#page-39-1)).

Two different estimators of the dependence function can be implemented. They are defined (on $0 \leq w \leq 1$) as follows.

method = "cfg" (Caperaa, Fougeres and Genest, 1997)

$$
\log(A_c(w)) = \frac{1}{n} \left\{ \sum_{i=1}^n \log(\max[(1-w)y_{i1}, wy_{i1}]) - (1-w)\sum_{i=1}^n y_{i1} - w\sum_{i=1}^n y_{i2} \right\}
$$

method = "pickands" (Pickands, 1981)

$$
A_p(w) = n \left\{ \sum_{i=1}^n \min \left(\frac{y_{i1}}{w}, \frac{y_{i2}}{1-w} \right) \right\}^{-1}
$$

Two variations on the estimator $A_p(\cdot)$ are also implemented. If the argument madj = 1, an adjustment given in Deheuvels (1991) is applied. If the argument mad $j = 2$, an adjustment given in Hall and Tajvidi (2000) is applied. These are marginal adjustments; they are only useful when empirical marginal estimation is used.

Let $A_n(\cdot)$ be any estimator of $A(\cdot)$. None of the estimators satisfy $\max(w, 1 - w) \le A_n(w) \le 1$ for all $0 \le w \le 1$. An obvious modification is

$$
A'_n(w) = \min(1, \max\{A_n(w), w, 1 - w\}).
$$

This modification is always implemented.

Convex estimators can be derived by taking the convex minorant, which can be achieved by setting convex to TRUE.

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Value

abvnonpar calculates or plots a non-parametric estimate of the dependence function of the bivariate extreme value distribution.

Note

I have been asked to point out that Hall and Tajvidi (2000) suggest putting a constrained smoothing spline on their modified Pickands estimator, but this is not done here.

References

Caperaa, P. Fougeres, A.-L. and Genest, C. (1997) A non-parametric estimation procedure for bivariate extreme value copulas. *Biometrika*, 84, 567–577.

Pickands, J. (1981) Multivariate extreme value distributions. *Proc. 43rd Sess. Int. Statist. Inst.*, 49, 859–878.

Deheuvels, P. (1991) On the limiting behaviour of the Pickands estimator for bivariate extremevalue distributions. *Statist. Probab. Letters*, 12, 429–439.

Hall, P. and Tajvidi, N. (2000) Distribution and dependence-function estimation for bivariate extremevalue distributions. *Bernoulli*, 6, 835–844.

See Also

[abvevd](#page-2-1), [amvnonpar](#page-8-1), [bvtcplot](#page-16-1), [fgev](#page-39-1)

Examples

bvdata <- rbvevd(100, dep = 0.7 , model = "log") $abronopar(seq(0, 1, length = 10)$, data = bvdata, convex = TRUE) abvnonpar(data = bvdata, method = "pick", plot = TRUE)

M1 <- fitted(fbvevd(bvdata, model = "log")) $abvevd(dep = M1['dep'']$, model = " $log"$, plot = TRUE) $abronpar(data = bvdata, add = TRUE, lty = 2)$

Description

Calculate the dependence function A for the multivariate logistic and multivariate asymmetric logistic models; plot the estimated function in the trivariate case.

Usage

```
amvevd(x = rep(1/d,d), dep, asy, model = c("log", "alog"), d = 3, plot =
   FALSE, col = heat.colors(12), blty = 0, grid = if(blty) 150 else 50,
   lower = 1/3, ord = 1:3, lab = as.character(1:3), lcex = 1)
```


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Details

Let $z = (z_1, \ldots, z_d)$ and $w = (w_1, \ldots, w_d)$. Any multivariate extreme value distribution can be written as

$$
G(z) = \exp\left\{-\left\{\sum_{j=1}^d y_j\right\} A\left(\frac{y_1}{\sum_{j=1}^d y_j}, \dots, \frac{y_d}{\sum_{j=1}^d y_j}\right)\right\}
$$

for some function A defined on the simplex $S_d = \{w \in R_+^d : \sum_{j=1}^d w_j = 1\}$, where

$$
y_i = \{1 + s_i(z_i - a_i)/b_i\}^{-1/s_i}
$$

for $1 + s_i(z_i - a_i)/b_i > 0$ and $i = 1, ..., d$, and where the (generalized extreme value) marginal parameters are given by (a_i, b_i, s_i) , $b_i > 0$. If $s_i = 0$ then y_i is defined by continuity.

A is called (by some authors) the dependence function. It follows that $A(w) = 1$ when w is one of the d vertices of S_d , and that A is a convex function with $\max(w_1, \ldots, w_d) \leq A(w) \leq 1$ for all w in S_d . The lower and upper limits of A are obtained under complete dependence and mutual independence respectively. A does not depend on the marginal parameters.

Value

A numeric vector of values. If plotting, the smallest evaluated function value is returned invisibly.

See Also

[amvnonpar](#page-8-1), [abvevd](#page-2-1), [rmvevd](#page-62-1), [image](#page-0-0)

Examples

```
amvevd(dep = 0.5, model = "log")s3pts \le matrix(rexp(30), nrow = 10, ncol = 3)
s3pts <- s3pts/rowSums(s3pts)
amvevd(s3pts, dep = 0.5, model = "log")## Not run: amvevd(dep = 0.05, model = "log", plot = TRUE, blty = 1)
amvevd(dep = 0.95, model = "log", plot = TRUE, lower = 0.94)asy \le list(.4, .1, .6, c(.3,.2), c(.1,.1), c(.4,.1), c(.2,.3,.2))
amvevd(s3pts, dep = 0.15, asy = asy, model = "alog")amvevd(dep = 0.15, asy = asy, model = "al", plot = TRUE, lower = 0.7)
```
amvnonpar *Non-parametric Estimates for Dependence Functions of the Multivariate Extreme Value Distribution*

Description

Calculate non-parametric estimates for the dependence function A of the multivariate extreme value distribution and plot the estimated function in the trivariate case.

Usage

```
amvnonpar(x = rep(1/d,d), data, d = 3, epman = FALSE, nshoc = NULL,madj = \theta, kmar = NULL, plot = FALSE, col = heat.colors(12),
   blty = 0, grid = if(blty) 150 else 50, lower = 1/3, ord = 1:3,
    lab = as.charAtacter(1:3), leex = 1)
```


Value

A numeric vector of estimates. If plotting, the smallest evaluated estimate is returned invisibly.

Note

The rows of data that contain missing values are not used in the estimation of the dependence structure, but every non-missing value is used in estimating the margins.

The dependence function of the multivariate extreme value distribution is defined in [amvevd](#page-6-1). The function [amvevd](#page-6-1) calculates and plots dependence functions of multivariate logistic and multivariate asymmetric logistic models.

The estimator plotted or calculated is a multivariate extension of the bivariate Pickands estimator defined in [abvnonpar](#page-4-1).

See Also

[amvevd](#page-6-1), [abvnonpar](#page-4-1), [fgev](#page-39-1)

Examples

```
s5pts \le matrix(rexp(50), nrow = 10, ncol = 5)
s5pts <- s5pts/rowSums(s5pts)
sdat <- rmvevd(100, dep = 0.6, model = "log", d = 5)
amvnonpar(s5pts, sdat, d = 5)
```

```
## Not run: amvnonpar(data = sdat, plot = TRUE)
## Not run: amvnonpar(data = sdat, plot = TRUE, ord = c(2,3,1), lab = LETTERS[1:3])
## Not run: amvevd(dep = 0.6, model = "log", plot = TRUE)
## Not run: amvevd(dep = 0.6, model = "log", plot = TRUE, blty = 1)
```


Description

Compute an analysis of deviance table for two or more nested evd objects.

Usage

```
## S3 method for class 'evd'
anova(object, object2, ..., half = FALSE)
```
Arguments

Value

An object of class c("anova", "data.frame"), with one row for each model, and the following five columns

Warning

Circumstances may arise such that the asymptotic distribution of the test statistic is not chi-squared. In particular, this occurs when the smaller model is constrained at the edge of the parameter space. It is up to the user recognize this, and to interpret the output correctly.

In some cases the asymptotic distribution is known to be one half of a chi-squared; you can set half = TRUE in these cases.

See Also

[fbvevd](#page-30-1), [fextreme](#page-37-1), [fgev](#page-39-1), [forder](#page-44-1)

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Examples

```
uvdata <- rgev(100, \text{loc} = 0.13, \text{scale} = 1.1, \text{shape} = 0.2)trend <- (-49:50)/100
M1 <- fgev(uvdata, nsloc = trend)
M2 <- fgev(uvdata)
M3 \leq fgev(uvdata, shape = 0)
anova(M1, M2, M3)
bvdata <- rbvevd(100, dep = 0.75, model = "log")
M1 <- fbvevd(bvdata, model = "log")
M2 \leq fbvevd(bvdata, model = "log", dep = 0.75)
M3 <- fbvevd(bvdata, model = "log", dep = 1)
anova(M1, M2)
anova(M1, M3, half = TRUE)
```
bvevd *Parametric Bivariate Extreme Value Distributions*

Description

Density function, distribution function and random generation for nine parametric bivariate extreme value models.

Usage

```
dbvevd(x, dep, asy = c(1, 1), alpha, beta, model = c("log", "alog",
    "hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
   mar1 = c(0, 1, 0), mar2 = mar1, log = FALSE)pbvevd(q, dep, asy = c(1, 1), alpha, beta, model = c("log", "alog","hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
   mar1 = c(0, 1, 0), mar2 = mar1, lower.tail = TRUE)
rbvevd(n, dep, asy = c(1, 1), alpha, beta, model = c("log", "alog","hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
   mar1 = c(0, 1, 0), mar2 = mar1)
```


Details

Define

$$
y_i = y_i(z_i) = \{1 + s_i(z_i - a_i)/b_i\}^{-1/s_i}
$$

for $1 + s_i(z_i - a_i)/b_i > 0$ and $i = 1, 2$, where the marginal parameters are given by mari = $(a_i, b_i, s_i), b_i > 0$. If $s_i = 0$ then y_i is defined by continuity.

In each of the bivariate distributions functions $G(z_1, z_2)$ given below, the univariate margins are generalized extreme value, so that $G(z_i) = \exp(-y_i)$ for $i = 1, 2$. If $1 + s_i(z_i - a_i)/b_i \leq 0$ for some $i = 1, 2$, the value z_i is either greater than the upper end point (if $s_i < 0$), or less than the lower end point (if $s_i > 0$), of the *i*th univariate marginal distribution.

$model = "log" (Gumbel, 1960)$

The bivariate logistic distribution function with parameter dep $= r$ is

$$
G(z_1, z_2) = \exp\left[-(y_1^{1/r} + y_2^{1/r})^r \right]
$$

where $0 < r \leq 1$. This is a special case of the bivariate asymmetric logistic model. Complete dependence is obtained in the limit as r approaches zero. Independence is obtained when $r = 1$.

$$
model = "alog" (Tawn, 1988)
$$

The bivariate asymmetric logistic distribution function with parameters dep = r and asy = (t_1, t_2) is

$$
G(z_1, z_2) = \exp \left\{ -(1 - t_1)y_1 - (1 - t_2)y_2 - [(t_1y_1)^{1/r} + (t_2y_2)^{1/r}]^r \right\}
$$

where $0 < r \le 1$ and $0 \le t_1, t_2 \le 1$. When $t_1 = t_2 = 1$ the asymmetric logistic model is equivalent to the logistic model. Independence is obtained when either $r = 1$, $t_1 = 0$ or $t_2 = 0$. Complete dependence is obtained in the limit when $t_1 = t_2 = 1$ and r approaches zero. Different limits occur when t_1 and t_2 are fixed and r approaches zero.

model = "hr" (Husler and Reiss, 1989)

The Husler-Reiss distribution function with parameter dep $= r$ is

$$
G(z_1, z_2) = \exp\left(-y_1 \Phi\{r^{-1} + \frac{1}{2}r[\log(y_1/y_2)]\} - y_2 \Phi\{r^{-1} + \frac{1}{2}r[\log(y_2/y_1)]\}\right)
$$

where $\Phi(\cdot)$ is the standard normal distribution function and $r > 0$. Independence is obtained in the limit as r approaches zero. Complete dependence is obtained as r tends to infinity.

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The bivariate negative logistic distribution function with parameter dep $= r$ is

$$
G(z_1, z_2) = \exp \left\{-y_1 - y_2 + [y_1^{-r} + y_2^{-r}]^{-1/r}\right\}
$$

where $r > 0$. This is a special case of the bivariate asymmetric negative logistic model. Independence is obtained in the limit as r approaches zero. Complete dependence is obtained as r tends to infinity. The earliest reference to this model appears to be Galambos (1975, Section 4).

model = "aneg log " (Joe, 1990)

The bivariate asymmetric negative logistic distribution function with parameters parameters dep $=$ r and asy $=(t_1, t_2)$ is

$$
G(z_1, z_2) = \exp \{-y_1 - y_2 + [(t_1y_1)^{-r} + (t_2y_2)^{-r}]^{-1/r}\}
$$

where $r > 0$ and $0 < t_1, t_2 \leq 1$. When $t_1 = t_2 = 1$ the asymmetric negative logistic model is equivalent to the negative logistic model. Independence is obtained in the limit as either r , t_1 or t_2 approaches zero. Complete dependence is obtained in the limit when $t_1 = t_2 = 1$ and r tends to infinity. Different limits occur when t_1 and t_2 are fixed and r tends to infinity. The earliest reference to this model appears to be Joe (1990), who introduces a multivariate extreme value distribution which reduces to $G(z_1, z_2)$ in the bivariate case.

 $model = "bilog" (Smith, 1990)$

The bilogistic distribution function with parameters alpha $=\alpha$ and beta $=\beta$ is

$$
G(z_1, z_2) = \exp\{-y_1 q^{1-\alpha} - y_2 (1-q)^{1-\beta}\}
$$

where $q = q(y_1, y_2; \alpha, \beta)$ is the root of the equation

$$
(1 - \alpha)y_1(1 - q)^{\beta} - (1 - \beta)y_2q^{\alpha} = 0,
$$

 $0 < \alpha, \beta < 1$. When $\alpha = \beta$ the bilogistic model is equivalent to the logistic model with dependence parameter dep = $\alpha = \beta$. Complete dependence is obtained in the limit as $\alpha = \beta$ approaches zero. Independence is obtained as $\alpha = \beta$ approaches one, and when one of α, β is fixed and the other approaches one. Different limits occur when one of α , β is fixed and the other approaches zero. A bilogistic model is fitted in Smith (1990), where it appears to have been first introduced.

model = "negbilog" (Coles and Tawn, 1994)

The negative bilogistic distribution function with parameters alpha = α and beta = β is

$$
G(z_1, z_2) = \exp \{-y_1 - y_2 + y_1 q^{1+\alpha} + y_2 (1-q)^{1+\beta} \}
$$

where $q = q(y_1, y_2; \alpha, \beta)$ is the root of the equation

$$
(1+\alpha)y_1q^{\alpha} - (1+\beta)y_2(1-q)^{\beta} = 0,
$$

 $\alpha > 0$ and $\beta > 0$. When $\alpha = \beta$ the negative bilogistic model is equivalent to the negative logistic model with dependence parameter dep $= 1/\alpha = 1/\beta$. Complete dependence is obtained in the limit as $\alpha = \beta$ approaches zero. Independence is obtained as $\alpha = \beta$ tends to infinity, and when one of α , β is fixed and the other tends to infinity. Different limits occur when one of α , β is fixed and the other approaches zero.

model = "ct" (Coles and Tawn, 1991)

The Coles-Tawn distribution function with parameters alpha = $\alpha > 0$ and beta = $\beta > 0$ is

$$
G(z_1, z_2) = \exp\{-y_1[1 - \text{Be}(q; \alpha + 1, \beta)] - y_2 \text{Be}(q; \alpha, \beta + 1)\}\
$$

where $q = \alpha y_2/(\alpha y_2 + \beta y_1)$ and Be(q; α, β) is the beta distribution function evaluated at q with shape1 = α and shape2 = β . Complete dependence is obtained in the limit as $\alpha = \beta$ tends to infinity. Independence is obtained as $\alpha = \beta$ approaches zero, and when one of α, β is fixed and the other approaches zero. Different limits occur when one of α , β is fixed and the other tends to infinity.

 $model = "amix" (Tawn, 1988)$

The asymmetric mixed distribution function with parameters alpha = α and beta = β has a dependence function with the following cubic polynomial form.

$$
A(t) = 1 - (\alpha + \beta)t + \alpha t^2 + \beta t^3
$$

where α and $\alpha+3\beta$ are non-negative, and where $\alpha+\beta$ and $\alpha+2\beta$ are less than or equal to one. These constraints imply that beta lies in the interval $[-0.5,0.5]$ and that alpha lies in the interval $[0,1.5]$, though alpha can only be greater than one if beta is negative. The strength of dependence increases for increasing alpha (for fixed beta). Complete dependence cannot be obtained. Independence is obtained when both parameters are zero. For the definition of a dependence function, see [abvevd](#page-2-1).

Value

dbvevd gives the density function, pbvevd gives the distribution function and rbvevd generates random deviates, for one of nine parametric bivariate extreme value models.

Note

The logistic and asymmetric logistic models respectively are simulated using bivariate versions of Algorithms 1.1 and 1.2 in Stephenson(2003). All other models are simulated using a root finding algorithm to simulate from the conditional distributions.

The simulation of the bilogistic and negative bilogistic models requires a root finding algorithm to evaluate q within the root finding algorithm used to simulate from the conditional distributions. The generation of bilogistic and negative bilogistic random deviates is therefore relatively slow (about 2.8 seconds per 1000 random vectors on a 450MHz PIII, 512Mb RAM).

The bilogistic and negative bilogistic models can be represented under a single model, using the integral of the maximum of two beta distributions (Joe, 1997).

The Coles-Tawn model is called the Dirichelet model in Coles and Tawn (1991).

References

Coles, S. G. and Tawn, J. A. (1991) Modelling extreme multivariate events. *J. Roy. Statist. Soc., B*, 53, 377–392.

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Galambos, J. (1975) Order statistics of samples from multivariate distributions. *J. Amer. Statist. Assoc.*, 70, 674–680.

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Gumbel, E. J. (1960) Distributions des valeurs extremes en plusieurs dimensions. *Publ. Inst. Statist. Univ. Paris*, 9, 171–173.

Husler, J. and Reiss, R.-D. (1989) Maxima of normal random vectors: between independence and complete dependence. *Statist. Probab. Letters*, 7, 283–286.

Joe, H. (1990) Families of min-stable multivariate exponential and multivariate extreme value distributions. *Statist. Probab. Letters*, 9, 75–81.

Joe, H. (1997) *Multivariate Models and Dependence Concepts*, London: Chapman & Hall.

Smith, R. L. (1990) Extreme value theory. In *Handbook of Applicable Mathematics* (ed. W. Ledermann), vol. 7. Chichester: John Wiley, pp. 437–471.

Stephenson, A. G. (2003) Simulating multivariate extreme value distributions of logistic type. *Extremes*, 6(1), 49–60.

Tawn, J. A. (1988) Bivariate extreme value theory: models and estimation. *Biometrika*, 75, 397– 415.

See Also

[abvevd](#page-2-1), [rgev](#page-50-1), [rmvevd](#page-62-1)

Examples

```
pbvevd(matrix(rep(0:4,2), ncol=2), dep = 0.7, model = "log")pbved(c(2,2), dep = 0.7, asy = c(0.6,0.8), model = "alog")pbved(c(1,1), dep = 1.7, model = "hr")margins \leq cbind(0, 1, seq(-0.5, 0.5, 0.1))
rbvevd(11, dep = 1.7, model = "hr", mar1 = margins)
rbvevd(10, dep = 1.2, model = "neglog", mar1 = c(10, 1, 1))
rbvevd(10, alpha = 0.7, beta = 0.52, model = "bilog")
dbvevd(c(0,0), dep = 1.2, asy = c(0.5,0.9), model = "aneglog")dbvevd(c(\emptyset, \emptyset), alpha = 0.75, beta = 0.5, model = "ct", log = TRUE)
dbvevd(c(\theta, \theta)), alpha = \theta.7, beta = 1.52, model = "negbilog")
```
bvtcplot *Bivariate Threshold Choice Plot*

Description

Produces a diagnostic plot to assist with threshold choice for bivariate data.

Usage

```
bvtcplot(x, spectral = FALSE, xlab, ylab, \ldots)
```
Arguments

Details

If spectral is FALSE (the default), produces a threshold choice plot as illustrated in Beirlant et al. (2004). With n non-missing bivariate observations, the integers $k = 1, \ldots, n - 1$ are plotted against the values $(k/n)r_{(n-k)}$, where $r_{(n-k)}$ is the $(n-k)$ th order statistic of the sum of the margins following empirical transformation to standard Frechet.

A vertical line is drawn at $k\emptyset$, where $k\emptyset$ is the largest integer for which the y-axis is above the value two. If spectral is FALSE, the largest k0 data points are used to plot an estimate of the spectal measure $H([0, w])$ versus w.

Value

A list is invisibly returned giving k0 and the values used to produce the plot.

References

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J. L. (2004) *Statistics of Extremes: Theory and Applications.*, Chichester, England: John Wiley and Sons.

See Also

[fbvpot](#page-34-1), [tcplot](#page-83-1)

Examples

```
## Not run: bvtcplot(lossalae)
## Not run: bvtcplot(lossalae, spectral = TRUE)
```


Calculate Conditional Copulas for Parametric Bivariate Extreme *Value Distributions*

Description

Conditional copula functions, conditioning on either margin, for nine parametric bivariate extreme value models.

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Usage

```
ccbvevd(x, mar = 2, dep, asy = c(1, 1), alpha, beta, model = c("log","alog", "hr", "neglog", "aneglog", "bilog", "negbilog", "ct",
    "amix"), lower.tail = TRUE)
```
Arguments

Details

The function calculates $P(U_1 \lt x_1 | U_2 = x_2)$, where (U_1, U_2) is a random vector with Uni $form(0,1)$ margins and with a dependence structure given by the specified parametric model. By default, the values of x_1 and x_1 are given by the first and second columns of the argument x. If mar = 1 then this is reversed.

If x has a third column x_3 of mode logical, then the function returns $P(U_1 < x_1 | U_2 = x_2, I = x_3)$, according to inference proceedures derived by Stephenson and Tawn (2004). See [fbvevd](#page-30-1). This requires numerical integration, and hence will be slower.

This function is mainly for internal use. It is used by $plot$, bvevd to calculate the conditional P-P plotting diagnostics.

Value

A numeric vector of probabilities.

References

Stephenson, A. G. and Tawn, J. A. (2004) Exploiting Occurence Times in Likelihood Inference for Componentwise Maxima. *Biometrika* 92(1), 213–217.

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See Also

[rbvevd](#page-12-1), [fbvevd](#page-30-1), [plot.bvevd](#page-67-1)

chiplot *Dependence Measure Plots*

Description

Plots of estimates of the dependence measures chi and chi-bar for bivariate data.

Usage

```
chiplot(data, nq = 100, qlim = NULL, which = 1:2, conf = 0.95, trunc =
    TRUE, spcases = FALSE, lty = 1, cilty = 2, col = 1, cicol = 1,
    xlim = c(0,1), ylim1 = c(-1,1), ylim2 = c(-1,1), main1 = "Chi Plot",
   main2 = "Chi Bar Plot", xlab = "Quantile", ylab1 = "Chi", ylab2 =
    "Chi Bar", ask = nb.fig < length(which) && dev.interactive(), ...)
```


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Details

These measures are explained in full detail in Coles, Heffernan and Tawn (1999). A brief treatment is also given in Section 8.4 of Coles(2001). A short summary is given as follows. We assume that the data are *iid* random vectors with common bivariate distribution function G , and we define the random vector (X, Y) to be distributed according to G.

The chi plot is a plot of q against empirical estimates of

$$
\chi(q) = 2 - \log(\Pr(F_X(X) < q, F_Y(Y) < q)) / \log(q)
$$

where F_X and F_Y are the marginal distribution functions, and where q is in the interval (0,1). The quantity $\chi(q)$ is bounded by

$$
2 - \log(2u - 1) / \log(u) \le \chi(q) \le 1
$$

where the lower bound is interpreted as -Inf for $q \leq 1/2$ and zero for $q = 1$. These bounds are reflected in the corresponding estimates.

The chi bar plot is a plot of q against empirical estimates of

$$
\bar{\chi}(q) = 2\log(1-q)/\log(\Pr(F_X(X) > q, F_Y(Y) > q)) - 1
$$

where F_X and F_Y are the marginal distribution functions, and where q is in the interval (0,1). The quantity $\bar{\chi}(q)$ is bounded by $-1 \leq \bar{\chi}(q) \leq 1$ and these bounds are reflected in the corresponding estimates.

Note that the empirical estimators for $\chi(q)$ and $\bar{\chi}(q)$ are undefined near $q = 0$ and $q = 1$. By default the function takes the limits of q so that the plots depicts all values at which the estimators are defined. This can be overridden by the argument qlim, which must represent a subset of the default values (and these can be determined using the component quantile of the invisibly returned list; see Value).

The confidence intervals within the plot assume that observations are independent, and that the marginal distributions are estimated exactly. The intervals are constructed using the delta method; this may lead to poor interval estimates near $q = 0$ and $q = 1$.

The function $\chi(q)$ can be interpreted as a quantile dependent measure of dependence. In particular, the sign of $\chi(q)$ determines whether the variables are positively or negatively associated at quantile level q. By definition, variables are said to be asymptotically independent when $\chi(1)$ (defined in the limit) is zero. For independent variables, $\chi(q) = 0$ for all q in (0,1). For perfectly dependent variables, $\chi(q) = 1$ for all q in (0,1). For bivariate extreme value distributions, $\chi(q) = 2(1 A(1/2)$) for all q in (0,1), where A is the dependence function, as defined in [abvevd](#page-2-1). If a bivariate threshold model is to be fitted (using [fbvpot](#page-34-1)), this plot can therefore act as a threshold identification plot, since e.g. the use of 95% marginal quantiles as threshold values implies that $\chi(q)$ should be approximately constant above $q = 0.95$.

The function $\bar{\chi}(q)$ can again be interpreted as a quantile dependent measure of dependence; it is most useful within the class of asymptotically independent variables. For asymptotically dependent variables (i.e. those for which $\chi(1) < 1$), we have $\bar{\chi}(1) = 1$, where $\bar{\chi}(1)$ is again defined in the limit. For asymptotically independent variables, $\bar{\chi}(1)$ provides a measure that increases with dependence strength. For independent variables $\bar{\chi}(q) = 0$ for all q in (0,1), and hence $\bar{\chi}(1) = 0$.

Value

A list with components quantile, chi (if 1 is in which) and chibar (if 2 is in which) is invisibly returned. The components quantile and chi contain those objects that were passed to the formal arguments x and y of matplot in order to create the chi plot. The components quantile and chibar contain those objects that were passed to the formal arguments x and y of matplot in order to create the chi-bar plot.

Author(s)

Jan Heffernan and Alec Stephenson

References

Coles, S. G., Heffernan, J. and Tawn, J. A. (1999) Dependence measures for extreme value analyses. *Extremes*, 2, 339–365.

Coles, S. G. (2001) *An Introduction to Statistical Modelling of Extreme Values*, London: Springer– Verlag.

See Also

[fbvevd](#page-30-1), [fbvpot](#page-34-1), [matplot](#page-0-0)

Examples

 $par(mfrow = c(1,2))$ smdat1 <- rbvevd(1000, dep = 0.6, model = "log") smdat2 <- r bvevd(1000, dep = 1, model = " log ") chiplot(smdat1) chiplot(smdat2)

clusters *Identify Clusters of Exceedences*

Description

Identify clusters of exceedences.

Usage

```
clusters(data, u, r = 1, ulow = -Inf, rlow = 1, cmax = FALSE, keep.namees= TRUE, plot = FALSE, xdata = seq(along = data), lvals = TRUE, lty =
   1, lwd = 1, pch = par("pch"), col = if(n > 250) NULL else "grey",
   xlab = "Index", ylab = "Data", ...)
```


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Arguments

Details

The clusters of exceedences are identified as follows. The first exceedence of the threshold initiates the first cluster. The first cluster then remains active until either r consecutive values fall below (or are equal to) the threshold, or until rlow consecutive values fall below (or are equal to) the lower threshold. The next exceedence of the threshold (if it exists) then initiates the second cluster, and so on. Missing values are allowed, in which case they are treated as falling below (or equal to) the threshold, but falling above the lower threshold.

Value

If cmax is FALSE (the default), a list with one component for each identified cluster. If cmax is TRUE, a numeric vector containing the cluster maxima. In any case, the returned object has an attribute acs, giving the average cluster size (where the cluster size is defined as the number of exceedences within a cluster), which will be NaN if there are no values above the threshold (and hence no clusters).

If plot is TRUE, the list of clusters, or vector of cluster maxima, is returned invisibly.

See Also

[exi](#page-26-1), [exiplot](#page-27-1)

Examples

```
clusters(portpirie, 4.2, 3)
clusters(portpirie, 4.2, 3, cmax = TRUE)
clusters(portpirie, 4.2, 3, 3.8, plot = TRUE)
clusters(portpirie, 4.2, 3, 3.8, plot = TRUE, lvals = FALSE)
tvu <- c(rep(4.2, 20), rep(4.1, 25), rep(4.2, 20))
clusters(portpirie, tvu, 3, plot = TRUE)
```
confint.evd *Calculate Confidence Intervals*

Description

Calculate profile and Wald confidence intervals of parameters in fitted models.

Usage

```
## S3 method for class 'evd'
confint(object, parm, level = 0.95, ...)## S3 method for class 'profile.evd'
confint(object, parm, level = 0.95, ...)
```


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Value

A matrix with two columns giving lower and upper confidence limits.

For profile confidence intervals, this function assumes that the profile trace is unimodal. If the profile trace is not unimodal then the function will give spurious results.

See Also

[profile.evd](#page-76-1)

Examples

```
m1 <- fgev(portpirie)
confint(m1)
## Not run: pm1 <- profile(m1)
## Not run: plot(pm1)
## Not run: confint(pm1)
```
evind.test *Perform Hypothesis Test Of Independence*

Description

Perform score and likelihood ratio tests of independence for bivariate data, assuming a logistic dependence model as the alternative.

Usage

```
evind.test(x, method = c("ratio", "score"), verbose = FALSE)
```
Arguments

Details

This simple function fits a stationary bivariate logistic model to the data and performs a hypothesis test of dep $= 1$ versus dep < 1 using the methodology in Tawn (1988). The null distributions for the printed test statistics are chi-squared on one df for the likelihood ratio test, and standard normal for the score test.

Value

An object of class "htest".

Tawn, J. A. (1988) Bivariate extreme value theory: models and estimation. *Biometrika*, 75, 397– 415.

See Also

[fbvevd](#page-30-1), [t.test](#page-0-0)

Examples

```
evind.test(sealevel)
evind.test(sealevel, method = "score")
```
evmc *Simulate Markov Chains With Extreme Value Dependence Structures*

Description

Simulation of first order Markov chains, such that each pair of consecutive values has the dependence structure of one of nine parametric bivariate extreme value distributions.

Usage

```
evmc(n, dep, asy = c(1,1), alpha, beta, model = c("log", "alog","hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
   margins = c("uniform","rweibull","frechet","gumbel"))
```


Value

A numeric vector of length n.

See Also

[marma](#page-58-1), [rbvevd](#page-12-1)

Examples

```
evmc(100, alpha = 0.1, beta = 0.1, model = "bilog")
evmc(100, dep = 10, model = "hr", margins = "gum")
```
exi *Estimates of the Extremal Index*

Description

Estimates of the extremal index.

Usage

exi(data, $u, r = 1$, $ulow = -Inf, rlow = 1$)

Arguments

Details

If r is a positive integer the extremal index is estimated using the inverse of the average cluster size, using the clusters of exceedences derived from [clusters](#page-21-1). If r is zero, an estimate based on inter-exceedance times is used (Ferro and Segers, 2003).

If there are no exceedances of the threshold, the estimate is NaN. If there is only one exceedance, the estimate is one.

Value

A single value estimating the extremal index.

References

Ferro, C. A. T. and Segers, J. (2003) Inference for clusters of extreme values. *JRSS B*, 65, 545–556.

See Also

[clusters](#page-21-1), [exiplot](#page-27-1)

Examples

```
exi(portpirie, 4.2, r = 3, ulow = 3.8)
tvu <- c(rep(4.2, 20), rep(4.1, 25), rep(4.2, 20))
exi(portpirie, tvu, r = 1)
exi(portpirie, tvu, r = 0)
```
exiplot *Plot Estimates of the Extremal Index*

Description

Plots estimates of the extremal index.

Usage

```
exiplot(data, tlim, r = 1, ulow = -Inf, rlow = 1, add = FALSE,
   nt = 100, lty = 1, xlab = "Threshold", ylab = "Ext. Index",ylim = c(0,1), \ldots)
```


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Details

The estimates are calculated using the function [exi](#page-26-1).

Value

A list with components x and y is invisibly returned. The first component contains the thresholds, the second contains the estimates.

See Also

[clusters](#page-21-1), [exi](#page-26-1)

Examples

```
sdat <- mar(100, \text{psi} = 0.5)tlim \leq quantile(sdat, probs = c(0.4, 0.9))
exiplot(sdat, tlim)
exiplot(sdat, tlim, r = 4, add = TRUE, lty = 2)
exiplot(sdat, tlim, r = 0, add = TRUE, lty = 4)
```
extreme *Distributions of Maxima and Minima*

Description

Density function, distribution function, quantile function and random generation for the maximum/minimum of a given number of independent variables from a specified distribution.

Usage

```
dextreme(x, densfun, distnfun, ..., distn, mlen = 1, largest = TRUE,
   log = FALSE)
pextreme(q, distnfun, ..., distn, mlen = 1, largest = TRUE,
   lower.tail = TRUE)
qextreme(p, quantfun, ..., distn, mlen = 1, largest = TRUE,
   lower.tail = TRUE)
rextreme(n, quantfun, ..., distn, mlen = 1, largest = TRUE)
```
Arguments

specified distribution. The density function must have a log argument (a simple wrapper can always be constructed to achieve this).

Value

dextreme gives the density function, pextreme gives the distribution function and qextreme gives the quantile function of the maximum/minimum of mlen independent variables from a specified distibution. rextreme generates random deviates.

See Also

[rgev](#page-50-1), [rorder](#page-66-1)

Examples

```
dextreme(2:4, dnorm, pnorm, mean = 0.5, sd = 1.2, mlen = 5)
dextreme(2:4, distn = "norm", mean = 0.5, sd = 1.2, mlen = 5)dextreme(2:4, distn = "exp", mlen = 2, largest = FALSE)
pextreme(2:4, distn = "exp", rate = 1.2, mlen = 2)
qextreme(seq(0.9, 0.6, -0.1), distn = "exp", rate = 1.2, mlen = 2)
rextreme(5, qgamma, shape = 1, mlen = 10)
p \leftarrow (1:9)/10pexp(qextreme(p, distn = "exp", rate = 1.2, mlen = 1), rate = 1.2)## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
```
failure *Failure Times*

Description

Failure times.

Usage

failure

Format

A vector containing 24 observations.

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Source

van Montfort, M. A. J. and Otten, A. (1978) On testing a shape parameter in the presence of a scale parameter. *Math. Operations Forsch. Statist., Ser. Statistics*, 9, 91–104.

Maximum-likelihood Fitting of Bivariate Extreme Value Distributions

Description

Fit models for one of nine parametric bivariate extreme value distributions, including linear modelling of the marginal location parameters, and allowing any of the parameters to be held fixed if desired.

Usage

```
fbvevd(x, model = c("log", "alog", "hr", "neglog", "aneglog", "bilog",
    "negbilog", "ct", "amix"), start, ..., sym = FALSE,
   nsloc1 = NULL, nsloc2 = NULL, cshape = cscale, cscale = cloc,
   cloc = FALSE, std.err = TRUE, corr = FALSE, method = "BFGS",warn.inf = TRUE)
```


Details

The dependence parameter names are one or more of dep, asy1, asy2, alpha and beta, depending on the model selected (see [rbvevd](#page-12-1)). The marginal parameter names are loc1, scale1 and shape1 for the first margin, and loc2, scale2 and shape2 for the second margin. If nsloc1 is not NULL, so that a linear model is implemented for the first marginal location parameter, the parameter names for the first margin are loc1, $loc1x1$, ..., $loc1xn$, scale and shape, where $x1$, ..., xn are the column names of nsloc1, so that loc1 is the intercept of the linear model, and loc1*x1*, . . . , loc1*xn* are the ncol(nsloc1) coefficients. When nsloc2 is not NULL, the parameter names for the second margin are constructed similarly.

It is recommended that the covariates within the linear models for the location parameters are (at least approximately) centered and scaled (i.e. that the columns of nsloc1 and nsloc2 are centered and scaled), particularly if automatic starting values are used, since the starting values for the associated parameters are then zero. If cloc is TRUE, both nsloc1 and nsloc2 must be identical, since a common linear model is then implemented on both margins.

If cshape is true, the models are constrained so that shape2 = shape1. The parameter shape2 is then taken to be specified, so that e.g. the common shape parameter can only be fixed at zero using shape1 = θ , since using shape2 = θ gives an error. Similar comments apply for cscale and cloc.

If sym is TRUE, the asymmetric logistic and asymmetric negative logistic models are constrained so that asy2 = asy1, and the Coles-Tawn model is constrained so that beta = alpha. The parameter asy2 or beta is then taken to be specified, so that e.g. the parameters asy1 and asy2 can only be fixed at 0.8 using asy1 = 0.8, since using $\text{asy2} = 0.8$ gives an error.

Bilogistic and negative bilogistic models constrained to symmetry are logistic and negative logistic models respectively. The (symmetric) mixed model (e.g. Tawn, 1998) can be obtained as a special case of the asymmetric logistic or asymmetric mixed models (see Examples).

The value Dependence given in the printed output is $2(1 - A(1/2))$, where A is the estimated dependence function (see [abvevd](#page-2-1)). It measures the strength of dependence, and lies in the interval [0,1]; at independence and complete dependence it is zero and one respectively (Coles, Heffernan and Tawn, 1999). See [chiplot](#page-19-1) for further information.

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Value

Returns an object of class c("bvevd","evd").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-2), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object.

The functions profile and profile2d can be used to obtain deviance profiles. The function anova compares nested models, and the function AIC compares non-nested models. The function plot produces diagnostic plots.

An object of class c("bvevd","evd") is a list containing the following components

More Details

If x is a data frame with a third column of mode logical, then the model is fitted using the likelihood derived by Stephenson and Tawn (2004). This is appropriate when each bivariate data point comprises componentwise maxima from some underlying bivariate process, and where the corresponding logical value denotes whether or not the maxima were caused by the same event within that process.

Under this scheme the diagnostic plots that are produced using plot are somewhat different to those described in [plot.bvevd](#page-67-1): the density, dependence function and quantile curves plots contain fitted functions for observations where the logical case is unknown, and the conditional P-P plots condition on both the logical case and the given margin (which requires numerical integration at each data point).

Artificial Constraints

For numerical reasons parameters are subject to artificial constraints. Specifically, these constraints are: marginal scale parameters not less than 0.01; dep not less than [0.1] [0.2] [0.05] in [logistic] [Husler-Reiss] [negative logistic] models; dep not greater than [10] [5] in [Husler-Reiss] [negative logistic] models; asy1 and asy2 not less than 0.001; alpha and beta not less than [0.1] [0.1] [0.001] in [bilogistic] [negative bilogistic] [Coles-Tawn] models; alpha and beta not greater than [0.999] [20] [30] in [bilogistic] [negative bilogistic] [Coles-Tawn] models.

Warning

The standard errors and the correlation matrix in the returned object are taken from the observed information, calculated by a numerical approximation. They must be interpreted with caution when either of the marginal shape parameters are less than -0.5 , because the usual asymptotic properties of maximum likelihood estimators do not then hold (Smith, 1985).

References

Coles, S. G., Heffernan, J. and Tawn, J. A. (1999) Dependence measures for extreme value analyses. *Extremes*, 2, 339–365.

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

Stephenson, A. G. and Tawn, J. A. (2004) Exploiting Occurence Times in Likelihood Inference for Componentwise Maxima. *Biometrika* 92(1), 213–217.

Tawn, J. A. (1988) Bivariate extreme value theory: models and estimation. *Biometrika*, 75, 397– 415.

See Also

[anova.evd](#page-11-1), [optim](#page-0-0), [plot.bvevd](#page-67-1), [profile.evd](#page-76-1), [profile2d.evd](#page-77-1), [rbvevd](#page-12-1)

Examples

```
bvdata <- rbvevd(100, dep = 0.6, model = "log", mar1 = c(1.2, 1.4, 0.4))
M1 <- fbvevd(bvdata, model = "log")
M2 \le fbvevd(bvdata, model = "log", dep = 0.75)
anova(M1, M2)
par(mfrow = c(2,2))plot(M1)
plot(M1, mar = 1)plot(M1, mar = 2)## Not run: par(mfrow = c(1,1))## Not run: M1P <- profile(M1, which = "dep")
## Not run: plot(M1P)
trend <- (-49:50)/100
rnd \le runif(100, min = -.5, max = .5)
fbvevd(bvdata, model = "log", nshol = trend)fbvevd(bvdata, model = "log", nsloc1 = trend, nsloc2 = data.frame(trend
= trend, random = rnd))
```
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```
fbvevd(bvdata, model = "log", nsloc1 = trend, nsloc2 = data.frame(trend
= trend, random = rnd), loc2random = 0)
bvdata \leq rbvevd(100, dep = 1, asy = c(0.5, 0.5), model = "anegl")
anlog <- fbvevd(bvdata, model = "anegl")
mixed <- fbvevd(bvdata, model = "anegl", dep = 1, sym = TRUE)
anova(anlog, mixed)
amixed <- fbvevd(bvdata, model = "amix")
mixed \leq fbvevd(bvdata, model = "amix", beta = 0)
anova(amixed, mixed)
```
fbvpot *Maximum-likelihood Fitting of Bivariate Extreme Value Distributions to Threshold Exceedances*

Description

Fit models for one of nine parametric bivariate extreme-value distributions using threshold exceedances, allowing any of the parameters to be held fixed if desired.

Usage

```
fbvpot(x, threshold, model = c("log", "bilog", "alog", "neglog",
    "negbilog", "aneglog", "ct", "hr", "amix"), likelihood =
   c("censored", "poisson"), start, ..., sym = FALSE, cshape =
   cscale, cscale = FALSE, std.err = TRUE, corr = FALSE, method =
    "BFGS", warn.inf = TRUE)
```


Details

For the "censored" method bivariate peaks over threshold models are fitted by maximizing the censored likelihood as given in e.g. Section 8.3.1 of Coles(2001). For the "poisson" method models are fitted using Equation 5.4 of Coles and Tawn (1991), see also Joe, Smith and Weissman (1992). This method is only available for models whose spectral measure does not contain point masses (see [hbvevd\)](#page-55-1). It is not recommended as in practice it can produce poor estimates.

For either likelihood the margins are modelled using a generalized Pareto distribution for points above the threshold and an empirical model for those below. For the "poisson" method data lying below both thresholds is not used. For the "censored" method the number of points lying below both thresholds is used, but the locations of the those points are not.

The dependence parameter names are one or more of dep, asy1, asy2, alpha and beta, depending on the model selected (see [rbvevd](#page-12-1)). The marginal parameter names are scale1 and shape1 for the first margin, and scale2 and shape2 for the second margin.

If cshape is true, the models are constrained so that shape2 = shape1. The parameter shape2 is then taken to be specified, so that e.g. the common shape parameter can only be fixed at zero using shape1 = θ , since using shape2 = θ gives an error. Similar comments apply for cscale.

If sym is TRUE, the asymmetric logistic and asymmetric negative logistic models are constrained so that asy2 = asy1, and the Coles-Tawn model is constrained so that beta = alpha. The parameter asy2 or beta is then taken to be specified, so that e.g. the parameters asy1 and asy2 can only be fixed at 0.8 using $asy1 = 0.8$, since using $asy2 = 0.8$ gives an error.

Bilogistic and negative bilogistic models constrained to symmetry are logistic and negative logistic models respectively. The (symmetric) mixed model (e.g. Tawn, 1998) can be obtained as a special case of the asymmetric logistic or asymmetric mixed models (see fbvevd).

For numerical reasons the parameters of each model are subject the artificial constraints given in [fbvevd](#page-30-1).

Value

Returns an object of class c("bvpot","evd").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-2), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object.
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The functions profile and profile2d can be used to obtain deviance profiles. The function anova compares nested models, and the function AIC compares non-nested models. There is currently no plot method available.

An object of class $c("b$ vpot", "evd") is a list containing the following components

Warning

The standard errors and the correlation matrix in the returned object are taken from the observed information, calculated by a numerical approximation. They must be interpreted with caution when either of the marginal shape parameters are less than -0.5 , because the usual asymptotic properties of maximum likelihood estimators do not then hold (Smith, 1985).

Author(s)

Chris Ferro and Alec Stephenson

References

Coles, S. G. (2001) *An Introduction to Statistical Modelling of Extreme Values*, London: Springer– Verlag.

Coles, S. G. and Tawn, J. A. (1991) Modelling multivariate extreme events. *J. R. Statist. Soc. B*, 53, 377–392.

Joe, H., Smith, R. L. and Weissman, I. (1992) Bivariate threshold methods for extremes. *J. R. Statist. Soc. B*, 54, 171–183.

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

See Also

[abvevd](#page-2-0), [anova.evd](#page-11-0), [fbvevd](#page-30-0), [optim](#page-0-0), [rbvevd](#page-12-0)

Examples

```
bvdata <- rbvevd(1000, dep = 0.5, model = "log")
u \leftarrow apply(bvdata, 2, quantile, probs = 0.9)M1 <- fbvpot(bvdata, u, model = "log")
M2 \leq fbvpot(bvdata, u, "log", dep = 0.5)
anova(M1, M2)
```
fextreme *Maximum-likelihood Fitting of Maxima and Minima*

Description

Maximum-likelihood fitting for the distribution of the maximum/minimum of a given number of independent variables from a specified distribution.

Usage

```
fextreme(x, start, densfun, distnfun, ..., distn, mlen = 1, largest =
   TRUE, std.err = TRUE, corr = FALSE, method = "Nelder-Mead")
```
Arguments

Density and distribution function of the specified distribution.

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Details

Maximization of the log-likelihood is performed. The estimated standard errors are taken from the observed information, calculated by a numerical approximation.

If the density and distribution functions are user defined, the order of the arguments must mimic those in R base (i.e. data first, parameters second). Density functions must have log arguments.

Value

Returns an object of class c("extreme","evd").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-0), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object. The function anova compares nested models.

An object of class $c("extreme", "evd")$ is a list containing at most the following components

See Also

[anova.evd](#page-11-0), [forder](#page-44-0), [optim](#page-0-0)

Examples

```
uvdata \le rextreme(100, qnorm, mean = 0.56, mlen = 365)
fextreme(uvdata, list(mean = 0, sd = 1), distn = "norm", mlen = 365)
fextreme(uvdata, list(rate = 1), distn = "exp", mlen = 365,
  method = "Brent", lower=0.01, upper=10)
fextreme(uvdata, list(scale = 1), shape = 1, distn = "gamma", mlen = 365,
  method = "Brent", lower=0.01, upper=10)
fextreme(uvdata, list(shape = 1, scale = 1), distn = "gamma", mlen = 365)
```


Maximum-likelihood Fitting of the Generalized Extreme Value Distri*bution*

Description

Maximum-likelihood fitting for the generalized extreme value distribution, including linear modelling of the location parameter, and allowing any of the parameters to be held fixed if desired.

Usage

```
fgev(x, start, ..., nsloc = NULL, prob = NULL, std.err = TRUE,corr = FALSE, method = "BFGS", warn.inf = TRUE)
```
Arguments

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Details

If prob is NULL (the default):

For stationary models the parameter names are loc, scale and shape, for the location, scale and shape parameters respectively. For non-stationary models, the parameter names are loc, loc*x1*, . . . , loc*xn*, scale and shape, where *x1*, . . . , *xn* are the column names of nsloc, so that loc is the intercept of the linear model, and $locx1$, ..., locxn are the ncol(nsloc) coefficients. If nsloc is a vector it is converted into a single column data frame with column name trend, and hence the associated trend parameter is named loctrend.

If prob $= p$ is a probability:

The fit is performed using a different parameterization. Let a , b and s denote the location, scale and shape parameters of the GEV distribution. For stationary models, the distribution is parameterized using (z_p, b, s) , where

$$
z_p = a - b/s(1 - (-\log(1 - p))^s)
$$

is such that $G(z_p) = 1 - p$, where G is the GEV distribution function. prob = p is therefore the probability in the upper tail corresponding to the quantile z_p . If prob is zero, then z_p is the upper end point $a - b/s$, and s is restricted to the negative (Weibull) axis. If prob is one, then z_p is the lower end point $a - b/s$, and s is restricted to the positive (Frechet) axis. The parameter names are quantile, scale and shape, for z_p , b and s respectively.

For non-stationary models the parameter z_p is again given by the equation above, but a becomes the intercept of the linear model for the location parameter, so that quantile replaces (the intercept) loc, and hence the parameter names are quantile, loc*x1*, . . . , loc*xn*, scale and shape, where *x1*, ..., *xn* are the column names of nsloc.

In either case:

For non-stationary fitting it is recommended that the covariates within the linear model for the location parameter are (at least approximately) centered and scaled (i.e.\ that the columns of nsloc are centered and scaled), particularly if automatic starting values are used, since the starting values for the associated parameters are then zero.

Value

Returns an object of class c("gev","uvevd","evd").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-0), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object.

The functions profile and profile2d are used to obtain deviance profiles for the model parameters. In particular, profiles of the quantile z_p can be calculated and plotted when prob = p. The function anova compares nested models. The function plot produces diagnostic plots.

An object of class c("gev", "uvevd", "evd") is a list containing at most the following components

Warning

The standard errors and the correlation matrix in the returned object are taken from the observed information, calculated by a numerical approximation. They must be interpreted with caution when the shape parameter is less than −0.5, because the usual asymptotic properties of maximum likelihood estimators do not then hold (Smith, 1985).

References

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

See Also

[anova.evd](#page-11-0), [optim](#page-0-0), [plot.uvevd](#page-73-0), [profile.evd](#page-76-0), [profile2d.evd](#page-77-0)

Examples

```
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
trend <- (-49:50)/100
M1 \leq fgev(uvdata, nsloc = trend, control = list(trace = 1))
M2 <- fgev(uvdata)
M3 \leq fgev(uvdata, shape = 0)
M4 \leq fgev(uvdata, scale = 1, shape = 0)anova(M1, M2, M3, M4)
par(mfrow = c(2,2))plot(M2)
## Not run: M2P <- profile(M2)
## Not run: plot(M2P)
rnd \le runif(100, min = -.5, max = .5)
fgev(uvdata, nsloc = data.frame(trend = trend, random = rnd))
fgev(uvdata, nsloc = data.frame(trend = trend, random = rnd), locrandom = \emptyset)
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
M1 \leq fgev(uvdata, prob = 0.1)
M2 \leq - fgev(uvdata, prob = 0.01)
```
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```
## Not run: M1P <- profile(M1, which = "quantile")
## Not run: M2P <- profile(M2, which = "quantile")
## Not run: plot(M1P)
## Not run: plot(M2P)
```
fgumbelx *Maximum-likelihood Fitting of the Maximum of Two Gumbel Distributions*

Description

Maximum-likelihood fitting for the maximum of two gumbel distributions, allowing any of the parameters to be held fixed if desired.

Usage

```
fgumbelx(x, start, ..., nslot = NULL, nshoc2 = NULL, std.err = TRUE,corr = FALSE, method = "BFGS", warn.inf = TRUE)
```
Arguments

Details

For stationary models the parameter names are loc1, scale1, loc2 and scale2 for the location and scale parameters of two Gumbel distributions, where loc2 must be greater or equal to loc1.

The likelihood may have multiple local optima and therefore may be difficult to fit properly; the default starting values use a moment based approach, however it is recommended that the user specify multiple different starting values and experiment with different optimization methods.

Using non-stationary models with nsloc1 and nsloc2 is not recommended due to the model complexity; the data also cannot be transformed back to stationarity so diagnostic plots will be misleading in this case.

Value

Returns an object of class c("gumbelx","evd").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-0), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object.

The functions profile and profile2d are used to obtain deviance profiles for the model parameters. The function anova compares nested models. The function plot produces diagnostic plots.

An object of class c("gumbelx","evd") is a list containing at most the following components

Warning

This function is experimental and involves optimizing over a potentially complex surface.

See Also

[fgev](#page-39-1), [optim](#page-0-0), [rgumbelx](#page-54-0)

Examples

```
uvdata <- rgumbelx(100, loc1 = 0, scale1 = 1, loc2 = 1, scale2 = 1)
fgumbelx(uvdata, loc1 = 0, scale1 = 1)
```


Description

Maximum-likelihood fitting for the distribution of a selected order statistic of a given number of independent variables from a specified distribution.

Usage

```
forder(x, start, densfun, distnfun, ..., distn, mlen = 1, j = 1,
   largest = TRUE, std.err = TRUE, corr = FALSE, method = "Nelder-Mead")
```
Arguments

Details

Maximization of the log-likelihood is performed. The estimated standard errors are taken from the observed information, calculated by a numerical approximation.

If the density and distribution functions are user defined, the order of the arguments must mimic those in R base (i.e. data first, parameters second). Density functions must have log arguments.

Value

Returns an object of class c("extreme","evd"). This class is defined in [fextreme](#page-37-0).

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-0), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object. The function anova compares nested models.

See Also

[anova.evd](#page-11-0), [fextreme](#page-37-0), [optim](#page-0-0)

Examples

```
uvd <- rorder(100, qnorm, mean = 0.56, mlen = 365, j = 2)
forder(uvd, list(mean = 0, sd = 1), distn = "norm", mlen = 365, j = 2)
forder(uvd, list(rate = 1), distn = "exp", mlen = 365, j = 2,
 method = "Brent", lower=0.01, upper=10)
forder(uvd, list(scale = 1), shape = 1, distn = "gamma", mlen = 365, j = 2,
 method = "Brent", lower=0.01, upper=10)
forder(uvd, list(shape = 1, scale = 1), distn = "gamma", mlen = 365, j = 2)
```
fox *Maximum Annual Flood Discharges of the Fox River*

Description

The fox data frame has 33 rows and 2 columns. The columns contain maximum annual flood discharges, in units of 1000 cubed feet per second, from the Fox River in Wisconsin, USA at Berlin (upstream) and Wrightstown (downstream), for the years 1918 to 1950. The row names give the years of observation.

Usage

fox

Format

This data frame contains the following columns:

berlin A numeric vector containing maximum annual flood discharges at Berlin (upstream).

wright A numeric vector containing maximum annual flood discharges at Wrightstown (downstream).

Source

Gumbel, E. J. and Mustafi, C. K. (1967) Some analytical properties of bivariate extremal distributions. *J. Amer. Statist. Assoc.*, 62, 569–588.

fpot *Peaks Over Threshold Modelling using the Generalized Pareto or Point Process Representation*

Description

Maximum-likelihood fitting for peaks over threshold modelling, using the Generalized Pareto or Point Process representation, allowing any of the parameters to be held fixed if desired.

Usage

```
fpot(x, threshold, model = c("gpd", "pp"), start, npp = length(x),
    cmax = FALSE, r = 1, ulow = -Inf, rlow = 1, mper = NULL, ...,
    std.err = TRUE, corr = FALSE, method = "BFGS", warn.inf = TRUE)
```
Arguments

Details

The exeedances over the threshold threshold (if cmax is FALSE) or the maxima of the clusters of exeedances (if cmax is TRUE) are (if model = "gpd") fitted to a generalized Pareto distribution (GPD) with location threshold. If model = "pp" the exceedances are fitted to a non-homogeneous Poisson process (Coles, 2001).

If mper is NULL (the default), the parameters of the model (if model $=$ "gpd") are scale and shape, for the scale and shape parameters of the GPD. If model = "pp" the parameters are loc, scale and shape. Under model = "pp" the parameters can be interpreted as parameters of the Generalized Extreme Value distribution, fitted to the maxima of npp random variables. In this case, the value of npp should be reasonably large.

For both characterizations, the shape parameters are equivalent. The scale parameter under the generalized Pareto characterization is equal to $b + s(u - a)$, where a, b and s are the location, scale and shape parameters under the Point Process characterization, and where u is the threshold.

If mper $= m$ is a positive value, then the generalized Pareto model is reparameterized so that the parameters are rlevel and shape, where rlevel is the m "period" return level, where "period" is defined via the argument npp.

The m "period" return level is defined as follows. Let G be the fitted generalized Pareto distribution function, with location threshold = u, so that $1 - G(z)$ is the fitted probability of an exceedance over $z > u$ given an exceedance over u. The fitted probability of an exceedance over $z > u$ is therefore $p(1 - G(z))$, where p is the estimated probabilty of exceeding u, which is given by the empirical proportion of exceedances. The m "period" return level z_m satisfies $p(1 - G(z_m)) =$ $1/(mN)$, where N is the number of points per period (multiplied by the estimate of the extremal index, if cluster maxima are fitted). In other words, z_m is the quantile of the fitted model that corresponds to the upper tail probability $1/(mN)$. If mper is infinite, then z_m is the upper end point, given by threshold minus scale/shape, and the shape parameter is then restricted to be negative.

Value

Returns an object of class c("pot","uvevd","pot").

The generic accessor functions [fitted](#page-0-0) (or [fitted.values](#page-0-0)), [std.errors](#page-39-0), [deviance](#page-0-0), [logLik](#page-0-0) and [AIC](#page-0-0) extract various features of the returned object.

The function profile can be used to obtain deviance profiles for the model parameters. In particular, profiles of the m period return level z_m can be calculated and plotted when mper = m. The function anova compares nested models. The function plot produces diagnostic plots.

An object of class c("pot","uvevd","evd") is a list containing the following components

- estimate A vector containing the maximum likelihood estimates.
- std.err A vector containing the standard errors.

Warning

The standard errors and the correlation matrix in the returned object are taken from the observed information, calculated by a numerical approximation. They must be interpreted with caution when the shape parameter is less than −0.5, because the usual asymptotic properties of maximum likelihood estimators do not then hold (Smith, 1985).

References

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

See Also

[anova.evd](#page-11-0), [optim](#page-0-0), [plot.uvevd](#page-73-0), [profile.evd](#page-76-0), [profile2d.evd](#page-77-0), [mrlplot](#page-59-0), [tcplot](#page-83-0)

Examples

```
uvdata \leq rgpd(100, loc = 0, scale = 1.1, shape = 0.2)
M1 <- fpot(uvdata, 1)
M2 \leq fpot(uvdata, 1, shape = 0)
anova(M1, M2)
par(mfrow = c(2,2))plot(M1)
## Not run: M1P <- profile(M1)
```

```
## Not run: plot(M1P)
M1 <- fpot(uvdata, 1, mper = 10)
M2 \leq fpot(uvdata, 1, mper = 100)
## Not run: M1P <- profile(M1, which = "rlevel", conf=0.975, mesh=0.1)
## Not run: M2P <- profile(M2, which = "rlevel", conf=0.975, mesh=0.1)
## Not run: plot(M1P)
## Not run: plot(M2P)
```
frechet *The Frechet Distribution*

Description

Density function, distribution function, quantile function and random generation for the Frechet distribution with location, scale and shape parameters.

Usage

 $dfrechet(x, loc=0, scale=1, shape=1, log = FALSE)$ pfrechet(q, loc=0, scale=1, shape=1, lower.tail = TRUE) qfrechet(p, loc=0, scale=1, shape=1, lower.tail = TRUE) rfrechet(n, loc=0, scale=1, shape=1)

Arguments

Details

The Frechet distribution function with parameters l oc = a, scale = b and shape = s is

$$
G(z) = \exp\left\{-\left(\frac{z-a}{b}\right)^{-s}\right\}
$$

for $z > a$ and zero otherwise, where $b > 0$ and $s > 0$.

Value

dfrechet gives the density function, pfrechet gives the distribution function, qfrechet gives the quantile function, and rfrechet generates random deviates.

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See Also

[rgev](#page-50-0), [rgumbel](#page-53-0), [rrweibull](#page-80-0)

Examples

```
dfrechet(2:4, 1, 0.5, 0.8)
pfrechet(2:4, 1, 0.5, 0.8)
qfrechet(seq(0.9, 0.6, -0.1), 2, 0.5, 0.8)
rfrechet(6, 1, 0.5, 0.8)
p \leftarrow (1:9)/10pfrechet(qfrechet(p, 1, 2, 0.8), 1, 2, 0.8)
## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
```
gev *The Generalized Extreme Value Distribution*

Description

Density function, distribution function, quantile function and random generation for the generalized extreme value (GEV) distribution with location, scale and shape parameters.

Usage

```
dgev(x, loc=0, scale=1, shape=0, log = FALSE)
pgev(q, loc=0, scale=1, shape=0, lower.tail = TRUE)
qgev(p, loc=0, scale=1, shape=0, lower.tail = TRUE)
rgev(n, loc=0, scale=1, shape=0)
```
Arguments

Details

The GEV distribution function with parameters $loc = a$, scale $= b$ and shape $= s$ is

$$
G(z) = \exp \left[-\{1 + s(z - a)/b\}^{-1/s} \right]
$$

for $1 + s(z - a)/b > 0$, where $b > 0$. If $s = 0$ the distribution is defined by continuity. If $1 + s(z - a)/b \leq 0$, the value z is either greater than the upper end point (if $s < 0$), or less than the lower end point (if $s > 0$).

The parametric form of the GEV encompasses that of the Gumbel, Frechet and reverse Weibull distributions, which are obtained for $s = 0$, $s > 0$ and $s < 0$ respectively. It was first introduced by Jenkinson (1955).

Value

dgev gives the density function, pgev gives the distribution function, qgev gives the quantile function, and rgev generates random deviates.

References

Jenkinson, A. F. (1955) The frequency distribution of the annual maximum (or minimum) of meteorological elements. *Quart. J. R. Met. Soc.*, 81, 158–171.

See Also

[fgev](#page-39-1), [rfrechet](#page-49-0), [rgumbel](#page-53-0), [rrweibull](#page-80-0)

Examples

dgev(2:4, 1, 0.5, 0.8) pgev(2:4, 1, 0.5, 0.8) qgev(seq(0.9, 0.6, -0.1), 2, 0.5, 0.8) rgev(6, 1, 0.5, 0.8) $p \leftarrow (1:9)/10$ pgev(qgev(p, 1, 2, 0.8), 1, 2, 0.8) ## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

gpd *The Generalized Pareto Distribution*

Description

Density function, distribution function, quantile function and random generation for the generalized Pareto distribution (GPD) with location, scale and shape parameters.

Usage

 $dgpd(x, loc=0, scale=1, shape=0, log = FALSE)$ pgpd(q, loc=0, scale=1, shape=0, lower.tail = TRUE) qgpd(p, loc=0, scale=1, shape=0, lower.tail = TRUE) rgpd(n, loc=0, scale=1, shape=0)

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Arguments

Details

The generalized Pareto distribution function (Pickands, 1975) with parameters $loc = a$, scale $= b$ and shape $= s$ is

$$
G(z) = 1 - \{1 + s(z - a)/b\}^{-1/s}
$$

for $1 + s(z - a)/b > 0$ and $z > a$, where $b > 0$. If $s = 0$ the distribution is defined by continuity.

Value

dgpd gives the density function, pgpd gives the distribution function, qgpd gives the quantile function, and rgpd generates random deviates.

References

Pickands, J. (1975) Statistical inference using extreme order statistics. *Annals of Statistics*, 3, 119– 131.

See Also

[fpot](#page-46-0), [rgev](#page-50-0)

Examples

dgpd(2:4, 1, 0.5, 0.8) pgpd(2:4, 1, 0.5, 0.8) qgpd(seq(0.9, 0.6, -0.1), 2, 0.5, 0.8) rgpd(6, 1, 0.5, 0.8) $p \leftarrow (1:9)/10$ pgpd(qgpd(p, 1, 2, 0.8), 1, 2, 0.8) ## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Description

Density function, distribution function, quantile function and random generation for the Gumbel distribution with location and scale parameters.

Usage

```
dgumbel(x, loc=0, scale=1, log = FALSE)
pgumbel(q, loc=0, scale=1, lower.tail = TRUE)
qgumbel(p, loc=0, scale=1, lower.tail = TRUE)
rgumbel(n, loc=0, scale=1)
```
Arguments

Details

The Gumbel distribution function with parameters $loc = a$ and $scale = b$ is

$$
G(z) = \exp\left\{-\exp\left[-\left(\frac{z-a}{b}\right)\right]\right\}
$$

for all real z, where $b > 0$.

Value

dgumbel gives the density function, pgumbel gives the distribution function, qgumbel gives the quantile function, and rgumbel generates random deviates.

See Also

[rfrechet](#page-49-0), [rgev](#page-50-0), [rrweibull](#page-80-0)

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Examples

```
dgumbel(-1:2, -1, 0.5)
pgumbel(-1:2, -1, 0.5)
qgumbel(seq(0.9, 0.6, -0.1), 2, 0.5)
rgumbel(6, -1, 0.5)
p \leftarrow (1:9)/10pgumbel(qgumbel(p, -1, 2), -1, 2)
## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
```
gumbelx *Maxima of Two Gumbel Distributions*

Description

Density function, distribution function, quantile function and random generation for the maxima of two Gumbel distributions, each with different location and scale parameters.

Usage

```
dgumbelx(x, loc1=0, scale1=1, loc2=0, scale2=1, log = FALSE)
pgumbelx(q, loc1=0, scale1=1, loc2=0, scale2=1, lower.tail = TRUE)
qgumbelx(p, interval, loc1=0, scale1=1, loc2=0, scale2=1, lower.tail = TRUE, ...)
rgumbelx(n, loc1=0, scale1=1, loc2=0, scale2=1)
```
Arguments

Value

dgumbelx gives the density function, pgumbelx gives the distribution function, qgumbelx gives the quantile function, and rgumbelx generates random deviates.

See Also

[fgev](#page-39-1), [rfrechet](#page-49-0), [rgumbel](#page-53-0), [rrweibull](#page-80-0), [uniroot](#page-0-0)

Examples

```
dgumbelx(2:4, 0, 1.1, 1, 0.5)
pgumbelx(2:4, 0, 1.1, 1, 0.5)
qgumbelx(seq(0.9, 0.6, -0.1), interval = c(0,10), 0, 1.2, 2, 0.5)
rgumbelx(6, 0, 1.1, 1, 0.5)
p \leftarrow (1:9)/10pgumbelx(qgumbelx(p, interval = c(0,10), 0, 0.5, 1, 2), 0, 0.5, 1, 2)## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
```


Description

Calculate or plot the density h of the spectral measure H on the interval $(0, 1)$, for nine parametric bivariate extreme value models.

Usage

```
hbvevd(x = 0.5, dep, asy = c(1,1), alpha, beta, model = c("log", "alog",
    "hr", "neglog", "aneglog", "bilog", "negbilog", "ct", "amix"),
    half = FALSE, plot = FALSE, add = FALSE, lty = 1, ...
```
Arguments

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Details

Any bivariate extreme value distribution can be written as

$$
G(z_1, z_2) = \exp\left[-\int_0^1 \max\{wy_1, (1-w)y_2\} H(dw)\right]
$$

for some function $H(\cdot)$ defined on [0, 1], satisfying

$$
\int_0^1 wH(dw) = \int_0^1 (1 - w)H(dw) = 1
$$

In particular, the total mass of H is two. The functions y_1 and y_2 are as defined in [abvevd](#page-2-0).

H is called the spectral measure, with density h on the interval $(0, 1)$.

Value

hbvevd calculates or plots the spectral density function h for one of nine parametric bivariate extreme value models, at specified parameter values.

Point Masses

For differentiable models H may have up to two point masses: at zero and one. Assuming that the model parameters are in the interior of the parameter space, we have the following. For the asymmetric logistic and asymmetric negative logistic models the point masses are of size 1-asy1 and 1-asy2 respectively. For the asymmetric mixed model they are of size 1-alpha-beta and 1-alpha-2*beta respectively. For all other models the point masses are zero.

At independence, H has point masses of size one at both zero and one. At complete dependence [a non-differentiable model] H has a single point mass of size two at $1/2$. In either case, h is zero everywhere.

See Also

[abvevd](#page-2-0), [fbvevd](#page-30-0), [rbvevd](#page-12-0), [plot.bvevd](#page-67-0)

Examples

```
hbvevd(dep = 2.7, model = "hr")
hbvevd(seq(0.25,0.5,0.75), dep = 0.3, asy = c(.7, .9), model = "alog")
hbvevd(alpha = 0.3, beta = 1.2, model = "negbi", plot = TRUE)
bvdata <- rbvevd(100, dep = 0.7, model = "log")
M1 <- fitted(fbvevd(bvdata, model = "log"))
hbvevd(dep = M1["dep"], model = "log", plot = TRUE)
```
Description

A numeric vector containing annual maximum wind speeds, in kilometers per hour, from 1941 to 1970 at Lisbon, Portugal.

Usage

lisbon

Format

A vector containing 30 observations.

Source

Tiago de Oliveira, J. (1997) *Statistical Analysis of Extremes.* Pendor.

lossalae *General Liability Claims*

Description

The lossalae data frame has 1500 rows and 2 columns. The columns contain the indemnity payment (loss), and the allocated loss adjustment expense (alae), both in USD. The latter is the additional expenses associated with the settlement of the claim (e.g. claims investigation expenses and legal fees).

The dataset also has an attribute called capped, which gives the row names of the indemnity payments that were capped at their policy limit.

Usage

lossalae

Format

This data frame contains the following columns:

Loss A numeric vector containing the indemnity payments.

ALAE A numeric vector containing the allocated loss adjustment expenses.

Source

Frees, E. W. and Valdez, E. A. (1998) Understanding relationships using copulas. *North American Actuarial Journal*, 2, 1–15.

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References

Klugman, S. A. and Parsa, R. (1999) Fitting bivariate loss distributions with copulas. *Insurance: Mathematics and Economics*, 24, 139–148.

Beirlant, J., Goegebeur, Y., Segers, J. and Teugels, J. L. (2004) *Statistics of Extremes: Theory and Applications.*, Chichester, England: John Wiley and Sons.

marma *Simulate MARMA(p,q) Processes*

Description

Simulation of MARMA(p,q) processes.

Usage

```
marma(n, p = 0, q = 0, psi, theta, init = rep(0, p), n.start = p,
    rand.gen = rfrechet, ...)
mar(n, p = 1, psi, init = rep(0, p), n.start = p, rand.gen =
    rfrechet, ...)
mma(n, q = 1, theta, rand.gen = rfrechet, ...)
```
Arguments

Details

A max autoregressive moving average process $\{X_k\}$, denoted by MARMA(p,q), is defined in Davis and Resnick (1989) as satisfying

 $X_k = \max\{\phi_1 X_{k-1}, \ldots, \phi_p X_{k-p}, \epsilon_k, \theta_1 \epsilon_{k-1}, \ldots, \theta_q \epsilon_{k-q}\}\$

where phi = (ϕ_1, \ldots, ϕ_p) and theta = $(\theta_1, \ldots, \theta_q)$ are non-negative vectors of parameters, and where $\{\epsilon_k\}$ is a series of *iid* random variables with a common distribution defined by rand.gen.

The functions mar and mma generate $MAR(p)$ and $MMA(q)$ processes respectively. A $MAR(p)$ process $\{X_k\}$ is equivalent to a MARMA(p, 0) process, so that

$$
X_k = \max\{\phi_1 X_{k-1}, \dots, \phi_p X_{k-p}, \epsilon_k\}.
$$

A MMA(q) process $\{X_k\}$ is equivalent to a MARMA(0, q) process, so that

$$
X_k = \max\{\epsilon_k, \theta_1 \epsilon_{k-1}, \ldots, \theta_q \epsilon_{k-q}\}.
$$

Value

A numeric vector of length n.

References

Davis, R. A. and Resnick, S. I. (1989) Basic properties and prediction of max-arma processes. *Adv. Appl. Prob.*, 21, 781–803.

See Also

[evmc](#page-25-0)

Examples

marma(100, $p = 1$, $q = 1$, $psi = 0.75$, theta = 0.65) mar(100, psi = 0.85, n.start = 20) mma(100, $q = 2$, theta = $c(0.75, 0.8)$)

Description

The empirical mean residual life plot.

Usage

```
mrlplot(data, tlim, pscale = FALSE, nt = max(100, length(data)), lty =
   c(2,1,2), col = 1, conf = 0.95, main = "Mean Residual Life Plot",
   xlab = "Threshold", ylab = "Mean Excess", ...)
```
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Arguments

Details

The empirical mean residual life plot is the locus of points

$$
\left(u, \frac{1}{n_u} \sum_{i=1}^{n_u} (x_{(i)} - u)\right)
$$

where $x_{(1)}, \ldots, x_{(n_u)}$ are the n_u observations that exceed the threshold u. If the exceedances of a threshold u_0 are generalized Pareto, the empirical mean residual life plot should be approximately linear for $u > u_0$.

The confidence intervals within the plot are symmetric intervals based on the approximate normality of sample means.

Value

A list with components x and y is invisibly returned. The components contain those objects that were passed to the formal arguments x and y of matplot in order to create the mean residual life plot.

Author(s)

Stuart Coles and Alec Stephenson

See Also

[fpot](#page-46-0), [matplot](#page-0-0), [tcplot](#page-83-0)

Examples

mrlplot(portpirie)

Description

Transforms to exponential margins under the GEV model.

Usage

 $mtransform(x, p, inv = FALSE, drp = FALSE)$

Arguments

Details

Let x_i denote a vector of observations for $i = 1, \ldots, n$. This function implements the transformation

$$
y_i = \{1 + s_i(x_i - a_i)/b_i\}_+^{-1/s_i}
$$

to each column of the matrix x.

The values (a_i, b_i, s_i) are contained in the ith row of the n by 3 matrix p. If p is a vector of length three, the parameters are the same for every $i = 1, \ldots, n$. Alternatively, p can be a list with d elements, in which case the jth element is used to transform the jth column of x.

This function is mainly for internal use. It is used by bivariate and multivariate routines to calculate marginal transformations.

Value

A numeric matrix or vector.

Description

Density function, distribution function and random generation for the multivariate logistic and multivariate asymmetric logistic models.

Usage

```
pmvevd(q, dep, asy, model = c("log", "alog"), d = 2, mar = <math>c(0,1,0)</math>,lower.tail = TRUE)rmvevd(n, dep, asy, model = c("log", "alog"), d = 2, mar = <math>c(0,1,0)</math>)dmvevd(x, dep, asy, model = c("log", "alog"), d = 2, mar = <math>c(0,1,0)</math>,log = FALSE)
```
Arguments

Details

Define

$$
y_i = y_i(z_i) = \{1 + s_i(z_i - a_i)/b_i\}^{-1/s_i}
$$

for $1 + s_i(z_i - a_i)/b_i > 0$ and $i = 1, \ldots, d$, where the marginal parameters are given by (a_i, b_i, s_i) , $b_i > 0$. If $s_i = 0$ then y_i is defined by continuity. Let $z = (z_1, z_2, \dots, z_d)$. In each of the multivariate distributions functions $G(z)$ given below, the univariate margins are generalized extreme value, so that $G(z_i) = \exp(-y_i)$ for $i = 1, \ldots, d$. If $1 + s_i(z_i - a_i)/b_i \le 0$ for some $i = 1, \ldots, d$, the value z_i is either greater than the upper end point (if $s_i < 0$), or less than the lower end point (if $s_i > 0$), of the *i*th univariate marginal distribution.

 $model = "log" (Gumbel, 1960)$

The d dimensional multivariate logistic distribution function with parameter dep $=r$ is

$$
G(z) = \exp\left\{-\left(\sum\nolimits_{i=1}^d y_i^{1/r}\right)^r\right\}
$$

where $0 < r < 1$. This is a special case of the multivariate asymmetric logistic model.

 $model = "alog" (Tawn, 1990)$

Let B be the set of all non-empty subsets of $\{1, \ldots, d\}$, let $B_1 = \{b \in B : |b| = 1\}$, where $|b|$ denotes the number of elements in the set b, and let $B(i) = \{b \in B : i \in b\}$. The d dimensional multivariate asymmetric logistic distribution function is

$$
G(z) = \exp\left\{-\sum\nolimits_{b \in B} \left[\sum\nolimits_{i \in b} (t_{i,b}y_i)^{1/r_b}\right]^{r_b}\right\},\,
$$

where the dependence parameters $r_b \in (0,1]$ for all $b \in B \setminus B_1$, and the asymmetry parameters $t_{i,b} \in [0,1]$ for all $b \in B$ and $i \in b$. The constraints $\sum_{b \in B_{(i)}} t_{i,b} = 1$ for $i = 1, \ldots, d$ ensure that the marginal distributions are generalized extreme value. Further constraints arise from the possible redundancy of asymmetry parameters in the expansion of the distribution form. Let $b_{-i_0} = \{i \in$ $b : i \neq i_0$. If $r_b = 1$ for some $b \in B \setminus B_1$ then $t_{i,b} = 0$ for all $i \in b$. Furthermore, if for some $b \in B \setminus B_1$, $t_{i,b} = 0$ for all $i \in b_{-i_0}$, then $t_{i_0,b} = 0$.

dep should be a vector of length $2^d - d - 1$ which contains $\{r_b : b \in B \setminus B_1\}$, with the order defined by the natural set ordering on the index. For example, for the trivariate model, dep $=$ $(r_{12}, r_{13}, r_{23}, r_{123})$. asy should be a list with $2^d - 1$ elements. Each element is a vector which corresponds to a set $b \in B$, containing $t_{i,b}$ for every integer $i \in b$. The elements should be given using the natural set ordering on the $b \in B$, so that the first d elements are vectors of length one corresponding to the sets $\{1\}, \ldots, \{d\}$, and the last element is a a vector of length d, corresponding to the set $\{1, \ldots, d\}$, asy must be constructed to ensure that all constraints are satisfied or an error will occur.

Value

pmvevd gives the distribution function, dmvevd gives the density function and rmvevd generates random deviates, for the multivariate logistic or multivariate asymmetric logistic model.

Note

Multivariate extensions of other bivariate models are more complex. A multivariate extension of the Husler-Reiss model exists, involving a multidimensional integral and one parameter for each

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bivariate margin. Multivariate extensions for the negative logistic model can be derived but are considerably more complex and appear to be less flexible. The "multivariate negative logistic model" often presented in the literature (e.g. Kotz *et al*, 2000) is not a valid distribution function and should not be used.

The logistic and asymmetric logistic models respectively are simulated using Algorithms 2.1 and 2.2 in Stephenson(2003b).

The density function of the logistic model is evaluated using the representation of Shi(1995). The density function of the asymmetric logistic model is evaluated using the representation given in Stephenson(2003a).

References

Gumbel, E. J. (1960) Distributions des valeurs extremes en plusieurs dimensions. *Publ. Inst. Statist. Univ. Paris*, 9, 171–173.

Kotz, S. and Balakrishnan, N. and Johnson, N. L. (2000) *Continuous Multivariate Distributions*, vol. 1. New York: John Wiley & Sons, 2nd edn.

Shi, D. (1995) Fisher information for a multivariate extreme value distribution. *Biometrika*, 82(3), 644–649.

Stephenson, A. G. (2003a) *Extreme Value Distributions and their Application*. Ph.D. Thesis, Lancaster University, Lancaster, UK.

Stephenson, A. G. (2003b) Simulating multivariate extreme value distributions of logistic type. *Extremes*, 6(1), 49–60.

Tawn, J. A. (1990) Modelling multivariate extreme value distributions. *Biometrika*, 77, 245–253.

See Also

[rbvevd](#page-12-0), [rgev](#page-50-0)

Examples

```
pmvevd(matrix(rep(0:4,5), ncol=5), dep = .7, model = "log", d = 5)
pmvcd(rep(4,5), dep = .7, model = "log", d = 5)rmvevd(10, dep = .7, model = "log", d = 5)
dmvevd(rep(-1,20), dep = .7, model = "log", d = 20, log = TRUE)
```

```
asy \leftarrow list(.4, .1, .6, c(.3,.2), c(.1,.1), c(.4,.1), c(.2,.3,.2))
pmvevd(rep(2,3), dep = c(.6,.5,.8,.3), asy = asy, model = "alog", d = 3)
asy <- list(.4, .0, .6, c(.3,.2), c(.1,.1), c(.4,.1), c(.2,.4,.2))
rmvevd(10, dep = c(.6,.5,.8,.3), asy = asy, model = "alog", d = 3)
dmvevd(rep(0,3), dep = c(.6,.5,.8,.3), asy = asy, model = "alog", d = 3)
```

```
asy <- list(0, 0, 0, 0, c(0,0), c(0,0), c(0,0), c(0,0), c(0,0), c(0,0),
 c(.2,.1,.2), c(.1,.1,.2), c(.3,.4,.1), c(.2,.2,.2), c(.4,.6,.2,.5))rmvevd(10, dep = .7, asy = asy, model = "alog", d = 4)
rmvevd(10, dep = c(rep(1,6), rep(.7,5)), asy = asy, model = "alog", d = 4)
```
Description

The ocmulgee data frame has 40 rows and 2 columns. The columns contain maximum annual flood discharges, in units of 1000 cubed feet per second, from the Ocmulgee River in Georgia, USA at Hawkinsville (upstream) and Macon (downstream), for the years 1910 to 1949. The row names give the years of observation.

Usage

ocmulgee

Format

This data frame contains the following columns:

hawk A numeric vector containing maximum annual flood discharges at Hawkinsville (upstream).

macon A numeric vector containing maximum annual flood discharges at Macon (downstream).

Source

Gumbel, E. J. and Goldstein, N. (1964) Analysis of empirical bivariate extremal distributions. *J. Amer. Statist. Assoc.*, 59, 794–816.

oldage *Oldest Ages for Swedish Males and Females*

Description

The oldage data frame has 66 rows and 2 columns. The columns contain the oldest ages at death for men and women in Sweden, for the period 1905–1970. The row names give the years of observation.

Usage

oldage

Format

This data frame contains the following columns:

men A numeric vector containing the oldest ages at death for men.

women A numeric vector containing the oldest ages at death for women.

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Source

Fransen, A. and Tiago de Oliveira, J. (1984) Statistical choice of univariate extreme models, part II, in *Statistical Extremes and Applications*, J. Tiago de Oliveira ed., 373–394, D. Reidel, Dordrect.

order *Distributions of Order Statistics*

Description

Density function, distribution function and random generation for a selected order statistic of a given number of independent variables from a specified distribution.

Usage

```
dorder(x, densfun, distnfun, ..., distn, mlen = 1, j = 1,
     largest = TRUE, log = FALSE)
porder(q, distnfun, ..., distn, mlen = 1, j = 1, largest = TRUE,
     lower.tail = TRUE)
rorder(n, quantfun, ..., distn, mlen = 1, j = 1, largest = TRUE)
```
Arguments

Value

dorder gives the density function, porder gives the distribution function and qorder gives the quantile function of a selected order statistic from a sample of size mlen, from a specified distibution. rorder generates random deviates.

See Also

[rextreme](#page-28-0), [rgev](#page-50-0)

Examples

```
dorder(2:4, dnorm, pnorm, mean = 0.5, sd = 1.2, mlen = 5, j = 2)
dorder(2:4, distn = "norm", mean = 0.5, sd = 1.2, mlen = 5, j = 2)
dorder(2:4, distn = "exp", mlen = 2, j = 2)
porder(2:4, distn = "exp", rate = 1.2, mlen = 2, j = 2)rorder(5, qgamma, shape = 1, mlen = 10, j = 2)
```
oxford *Annual Maximum Temperatures at Oxford*

Description

A numeric vector containing annual maximum temperatures, in degrees Fahrenheit, from 1901 to 1980 at Oxford, England.

Usage

oxford

Format

A vector containing 80 observations.

Source

Tabony, R. C. (1983) Extreme value analysis in meteorology. *The Meteorological Magazine* 112, 77–98.

plot.bvevd *Plot Diagnostics for a Bivariate EVD Object*

Description

Six plots (selectable by which) are currently provided: two conditional P-P plots (1,2), conditioning on each margin, a density plot (3), a dependence function plot (4), a quantile curves plot (5) and a spectral density plot (6). Plot diagnostics for the generalized extreme value margins (selectable by mar and which) are also available.

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Usage

```
## S3 method for class 'bvevd'
plot(x, mar = 0, which = 1:6, main, ask = nb.fig <length(which) && dev.interactive(), ci = TRUE, cilwd = 1,
   a = 0, grid = 50, legend = TRUE, nplty = 2, blty = 3, method = "cfg",
   convex = FALSE, rev = FALSE, p = seq(0.75, 0.95, 0.05),
   mint = 1, half = FALSE, ...)
```
Arguments

Details

In all plots we assume that the fitted model is stationary. For non-stationary models the data are transformed to stationarity. The plot then corresponds to the distribution obtained when all covariates are zero. In particular, the density and quanitle curves plots will not plot the original data for non-stationary models.

A conditional P-P plot is a P-P plot for the condition distribution function of a bivariate evd object. Let $G(.)$ be the conditional distribution of the first margin given the second, under the fitted model. Let z_1, \ldots, z_m be the data used in the fitted model, where $z_j = (z_{1j}, z_{2j})$ for $j = 1, \ldots, m$. The plot that (by default) is labelled Conditional Plot Two, conditioning on the second margin, consists of the points

$$
\{(p_i,c_i), i=1,\ldots,m\}
$$

where p_1, \ldots, p_m are plotting points defined by [ppoints](#page-0-0) and c_i is the *i*th largest value from the sample $\{G(z_{i1}|z_{i2}), j = 1, \ldots, m\}$. The margins are reversed for Conditional Plot One, so that $G(.|.)$ is the conditional distribution of the second margin given the first.

See Also

[plot.uvevd](#page-73-0), [contour](#page-0-0), [jitter](#page-0-0), [abvnonpar](#page-4-0), [qcbvnonpar](#page-78-0)

Examples

```
bvdata \leq rbvevd(100, dep = 0.6, model = "log")
M1 \leq - fbvevd(bvdata, model = "log")
## Not run: par(mfrow = c(2,2))## Not run: plot(M1, which = 1:5)
## Not run: plot(M1, mar = 1)
## Not run: plot(M1, mar = 2)
```
plot.bvpot *Plot Diagnostics for a Bivariate POT EVD Object*

Description

Four plots (selectable by which) are currently provided: a density plot (1), a dependence function plot (2), a quantile curves plot (3) and a spectral density plot (4). Plot diagnostics for the generalized Pareto peaks-over-threshold margins (selectable by mar and which) are also available.

Usage

```
## S3 method for class 'bvpot'
plot(x, mar = 0, which = 1:4, main, ask = nb.fig <length(which) && dev.interactive(), grid = 50, above = FALSE,
    levels = NULL, tlty = 1, blty = 3, rev = FALSE, p = seq(0.75,0.95, 0.05), half = FALSE, ...)
```
plot.profile.evd 71

Arguments

See Also

[plot.bvevd](#page-67-0), [contour](#page-0-0), [abvnonpar](#page-4-0), [qcbvnonpar](#page-78-0), [hbvevd](#page-55-0)

Examples

```
bvdata \leq rbvevd(500, dep = 0.6, model = "log")
M1 <- fbvpot(bvdata, threshold = c(\emptyset, \emptyset), model = "log")
## Not run: plot(M1)
## Not run: plot(M1, mar = 1)
## Not run: plot(M1, mar = 2)
```
plot.profile.evd *Plot Profile Log-likelihoods*

Description

Displays profile log-likelihoods from a model profiled with [profile.evd](#page-76-0).

Usage

```
## S3 method for class 'profile.evd'
plot(x, which = names(x), main = NULL,ask = nb.fig < length(which) && dev.interactive(), ci = 0.95,
    \text{clty} = 2, \ldots)
```
Arguments

Value

Profile devainces are plotted for each parameter in which. For calculation of profile confidence intervals, use the [confint.profile.evd](#page-23-0) function.

Warning

The profile confidence intervals may not have confidence coefficient ci, because the usual asymptotic properties of maximum likelihood estimators may not hold. For the GEV model, the usual asymptotic properties hold when the shape parameter is greater than −0.5 (Smith, 1985).

References

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

See Also

[confint.profile.evd](#page-23-0), [plot.profile2d.evd](#page-72-0), [profile.evd](#page-76-0), [profile2d.evd](#page-77-0)

Examples

```
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
M1 <- fgev(uvdata)
## Not run: M1P <- profile(M1)
## Not run: par(mfrow = c(2,2))
```
plot.profile2d.evd 73

```
## Not run: cint <- plot(M1P, ci = c(0.95, 0.99))
## Not run: cint
```
plot.profile2d.evd *Plot Joint Profile Log-likelihoods*

Description

Displays an image plot of the joint profile log-likelihood from a model profiled with [profile.evd](#page-76-0) and [profile2d.evd](#page-77-0).

Usage

S3 method for class 'profile2d.evd' plot(x, main = NULL, $ci = c(0.5, 0.8, 0.9, 0.95, 0.975, 0.99, 0.995),$ $col = heat.colors(8)$, intpts = 75, xaxs = "r", yaxs = "r", ...)

Arguments

Warning

The sets represented by different colours may not be confidence sets with confidence coefficients ci, because the usual asymptotic properties of maximum likelihood estimators may not hold. For the GEV model, the usual asymptotic properties hold when the shape parameter is greater than -0.5 (Smith, 1985).

References

Smith, R. L. (1985) Maximum likelihood estimation in a class of non-regular cases. *Biometrika*, 72, 67–90.

See Also

[plot.profile.evd](#page-70-0), [profile.evd](#page-76-0), [profile2d.evd](#page-77-0)

Examples

```
uvdata <- rgev(100, 10c = 0.13, scale = 1.1, shape = 0.2)M1 <- fgev(uvdata)
## Not run: M1P <- profile(M1)
## Not run: M1JP <- profile2d(M1, M1P, which = c("scale", "shape"))
## Not run: plot(M1JP)
```
plot.uvevd *Plot Diagnostics for a Univariate EVD Object*

Description

Four plots (selectable by which) are currently provided: a P-P plot, a Q-Q plot, a density plot and a return level plot.

Usage

```
## S3 method for class 'uvevd'
plot(x, which = 1:4, main, ask = nb.fig <length(which) && dev.interactive(), ci = TRUE, cilwd = 1,
    a = 0, adjust = 1, jitter = FALSE, nplty = 2, ...)
## S3 method for class 'gumbelx'
plot(x, interval, which = 1:4, main, ask = nb.fig <length(which) && dev.interactive(), ci = TRUE, cilwd = 1,
    a = 0, adjust = 1, jitter = FALSE, nplty = 2, ...)
```
Arguments

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Details

The following discussion assumes that the fitted model is stationary. For non-stationary generalized extreme value models the data are transformed to stationarity. The plot then corresponds to the distribution obtained when all covariates are zero.

The P-P plot consists of the points

$$
\{(G_n(z_i), G(z_i)), i = 1, \ldots, m\}
$$

where G_n is the empirical distribution function (defined using [ppoints](#page-0-0)), G is the model based estimate of the distribution (generalized extreme value or generalized Pareto), and z_1, \ldots, z_m are the data used in the fitted model, sorted into ascending order.

The Q-Q plot consists of the points

$$
\{(G^{-1}(p_i), z_i), i = 1, \ldots, m\}
$$

where G^{-1} is the model based estimate of the quantile function (generalized extreme value or generalized Pareto), p_1, \ldots, p_m are plotting points defined by [ppoints](#page-0-0), and z_1, \ldots, z_m are the data used in the fitted model, sorted into ascending order.

The return level plot for generalized extreme value models is defined as follows.

Let G be the generalized extreme value distribution function, with location, scale and shape parameters a, b and s respectively. Let z_t be defined by $G(z_t) = 1 - 1/t$. In common terminology, z_t is the return level associated with the return period t .

Let $y_t = -1/\log(1 - 1/t)$. It follows that

$$
z_t = a + b(y_t^s - 1)/s.
$$

When $s = 0$, z_t is defined by continuity, so that

$$
z_t = a + b \log(y_t).
$$

The curve within the return level plot is z_t plotted against y_t on a logarithmic scale, using maximum likelihood estimates of (a, b, s) . If the estimate of s is zero, the curve will be linear. For large values of t, y_t is approximately equal to the return period t. It is usual practice to label the x-axis as the return period.

The points on the plot are

$$
\{(-1/\log(p_i), z_i), i = 1, \ldots, m\}
$$

where p_1, \ldots, p_m are plotting points defined by [ppoints](#page-0-0), and z_1, \ldots, z_m are the data used in the fitted model, sorted into ascending order. For a good fit the points should lie "close" to the curve.

The return level plot for peaks over threshold models is defined as follows.

Let G be the generalized Pareto distribution function, with location, scale and shape parameters u , b and s respectively, where u is the model threshold. Let z_m denote the m period return level (see [fpot](#page-46-0) and the notation therein). It follows that

$$
z_m = u + b((pmN)^s - 1)/s.
$$

When $s = 0$, z_m is defined by continuity, so that

$$
z_m = u + b \log(pmN).
$$

The curve within the return level plot is z_m plotted against m on a logarithmic scale, using maximum likelihood estimates of (b, s, p) . If the estimate of s is zero, the curve will be linear.

The points on the plot are

$$
\{(1/(pN(1-p_i)), z_i), i=1,\ldots,m\}
$$

where p_1, \ldots, p_m are plotting points defined by [ppoints](#page-0-0), and z_1, \ldots, z_m are the data used in the fitted model, sorted into ascending order. For a good fit the points should lie "close" to the curve.

See Also

[plot.bvevd](#page-67-0), [density](#page-0-0), [jitter](#page-0-0), [rug](#page-0-0), [ppoints](#page-0-0)

Examples

```
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
M1 <- fgev(uvdata)
## Not run: par(mfrow = c(2,2))## Not run: plot(M1)
uvdata <- rgpd(100, loc = 0, scale = 1.1, shape = 0.2)
M1 <- fpot(uvdata, 1)
## Not run: par(mfrow = c(2,2))
## Not run: plot(M1)
```
portpirie *Annual Maximum Sea Levels at Port Pirie*

Description

A numeric vector containing annual maximum sea levels, in metres, from 1923 to 1987 at Port Pirie, South Australia.

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Usage

portpirie

Format

A vector containing 65 observations.

Source

Tawn, J. A. (1993) Extreme sea-levels, in *Statistics in the Environment*, 243–263, eds. V. Barnett and F. Turkman, Wiley.

References

Coles, S. G. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

profile.evd *Method for Profiling EVD Objects*

Description

Calculate profile traces for fitted models.

Usage

```
## S3 method for class 'evd'
profile(fitted, which = names(fitted$estimate), conf = 0.999,
   mesh = fitted$std.err[which]/4, xmin = rep(-Inf, length(which)),
   xmax = rep(Inf, length(which)), convergence = FALSE, method = "BFGS",
   control = list(maxit = 500), ...
```
Arguments

Value

An object of class "profile.evd", which is a list with an element for each parameter being profiled. The elements are matrices. The first column contains the values of the profiled parameter. The second column contains profile deviances. The remaining columns contain the constrained maximum likelihood estimates for the remaining model parameters. For calculation of profile confidence intervals, use the [confint.profile.evd](#page-23-0) function.

See Also

[confint.profile.evd](#page-23-0), [profile2d.evd](#page-77-0), [plot.profile.evd](#page-70-0)

Examples

```
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
M1 <- fgev(uvdata)
## Not run: M1P <- profile(M1)
## Not run: par(mfrow = c(2,2))## Not run: cint <- plot(M1P)
## Not run: cint
```
profile2d.evd *Method for Profiling EVD Objects*

Description

Calculate joint profile traces for fitted models.

Usage

```
## S3 method for class 'evd'
profile2d(fitted, prof, which, pts = 20, convergence =
   FALSE, method = "Nelder-Mead", control = list(maxit = 5000), ...
```
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Arguments

Value

An object of class "profile2d.evd", which is a list with three elements. The first element, a matrix named trace, has the same structure as the elements of an object of class "profile.evd". The last two elements give the distinct values used for each profiled parameter in which.

See Also

[profile.evd](#page-76-0), [plot.profile2d.evd](#page-72-0)

Examples

```
uvdata <- rgev(100, loc = 0.13, scale = 1.1, shape = 0.2)
M1 <- fgev(uvdata)
## Not run: M1P <- profile(M1)
## Not run: M1JP <- profile2d(M1, M1P, which = c("scale", "shape"))
## Not run: plot(M1JP)
```
qcbvnonpar *Non-parametric Estimates for Bivariate Quantile Curves*

Description

Calculate or plot non-parametric estimates for quantile curves of bivariate extreme value distributions.

Usage

```
qcbvnonpar(p = \text{seq}(0.75, 0.95, 0.05), data, epmar = FALSE, nsloc1 =
    NULL, nsloc2 = NULL, mint = 1, method = c("cfg", "pickands",
    "tdo"), convex = FALSE, madj = 0, kmar = NULL, plot = FALSE,
    add = FALSE, 1ty = 1, 1wd = 1, col = 1, xlim = range(data[, 1],na.rm = TRUE), ylim = range(data[, 2], na.rm = TRUE), xlab =collnames(data)[1], ylab = colnames(data)[2], ...
```
Arguments

Details

Let G be a fitted bivariate distribution function with margins G_1 and G_2 . A quantile curve for a fitted distribution function G at lower tail probability p is defined by

$$
Q(G, p) = \{ (y_1, y_1) : G(y_1, y_2) = p \}.
$$

For bivariate extreme value distributions, it consists of the points

$$
\left\{G_1^{-1}(p_1), G_2^{-1}(p_2))\right\}
$$

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where $p_1 = p^{t/A(t)}$ and $p_2 = p^{(1-t)/A(t)}$, with A being the estimated dependence function defined in [abvevd](#page-2-0), and where t lies in the interval $[0, 1]$.

By default the margins G_1 and G_2 are modelled using estimated generalized extreme value distributions. For non-stationary generalized extreme value margins the plotted data are transformed to stationarity, and the plot corresponds to the distribution obtained when all covariates are zero.

If epmar is TRUE, empirical transformations are used in preference to generalized extreme value models. Note that the marginal empirical quantile functions are evaluated using [quantile](#page-0-0), which linearly interpolates between data points, hence the curve will not be a step function.

The idea behind the argument mint $=m$ is that if G is fitted to a dataset of componentwise maxima, and the underlying observations are *iid* distributed according to F, then if m is the size of the blocks over which the maxima were taken, approximately $F^m = G$, leading to $Q(F, p) = Q(G, p^m)$.

Value

qcbvnonpar calculates or plots non-parametric quantile curve estimates for bivariate extreme value distributions. If p has length one it returns a two column matrix giving points on the curve, else it returns a list of such matrices.

See Also

[abvevd](#page-2-0), [abvnonpar](#page-4-0), [plot.bvevd](#page-67-0)

Examples

bvdata \leq rbvevd(100, dep = 0.7, model = "log") $qcbvnonpar(c(0.9, 0.95), data = bvdata, plot = TRUE)$ $qcbvnonpar(c(0.9, 0.95), data = bvdata, epmar = TRUE, plot = TRUE)$

rweibull *The Reverse Weibull Distribution*

Description

Density function, distribution function, quantile function and random generation for the reverse (or negative) Weibull distribution with location, scale and shape parameters.

Usage

```
drweibull(x, loc=0, scale=1, shape=1, log = FALSE)
prweibull(q, loc=0, scale=1, shape=1, lower.tail = TRUE)
qrweibull(p, loc=0, scale=1, shape=1, lower.tail = TRUE)
rrweibull(n, loc=0, scale=1, shape=1)
dnweibull(x, loc=0, scale=1, shape=1, log = FALSE)
pnweibull(q, loc=0, scale=1, shape=1, lower.tail = TRUE)
qnweibull(p, loc=0, scale=1, shape=1, lower.tail = TRUE)
rnweibull(n, loc=0, scale=1, shape=1)
```
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Arguments

Details

The reverse (or negative) Weibull distribution function with parameters $loc = a$, scale $= b$ and shape $= s$ is

$$
G(z) = \exp\left\{-\left[-\left(\frac{z-a}{b}\right)\right]^s\right\}
$$

for $z < a$ and one otherwise, where $b > 0$ and $s > 0$.

Value

drweibull and dnweibull give the density function, prweibull and pnweibull give the distribution function, qrweibull and qnweibull give the quantile function, rrweibull and rnweibull generate random deviates.

Note

Within extreme value theory the reverse Weibull distibution (also known as the negative Weibull distribution) is often referred to as the Weibull distribution. We make a distinction to avoid confusion with the three-parameter distribution used in survival analysis, which is related by a change of sign to the distribution given above.

See Also

[rfrechet](#page-49-0), [rgev](#page-50-0), [rgumbel](#page-53-0)

Examples

```
drweibull(-5:-3, -1, 0.5, 0.8)
prweibull(-5:-3, -1, 0.5, 0.8)
qrweibull(seq(0.9, 0.6, -0.1), 2, 0.5, 0.8)
rrweibull(6, -1, 0.5, 0.8)
p \leftarrow (1:9)/10prweibull(qrweibull(p, -1, 2, 0.8), -1, 2, 0.8)
## [1] 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
```
Description

A numeric vector containing maximum annual flood discharges, in units of 1000 cubic feet per second, of the North Saskachevan River at Edmonton, over a period of 47 years. Unfortunately, the data are ordered from largest to smallest.

Usage

sask

Format

A vector containing 47 observations.

Source

van Montfort, M. A. J. (1970) On testing that the distribution is of type I when type II is the alternative. *J. Hydrology*, 11, 421–427.

sealevel *Annual Sea Level Maxima at Dover and Harwich*

Description

The sealevel data frame has 81 rows and 2 columns. The columns contain annual sea level maxima from 1912 to 1992 at Dover and Harwich respectively, two sites on the coast of Britain. The row names give the years of observation. There are 39 missing values.

Usage

sealevel

Format

This data frame contains the following columns:

dover A numeric vector containing annual sea level maxima at Dover, including 9 missing values.

harwich A numeric vector containing sea annual level maxima at Harwich, including 30 missing values.

Source

Coles, S. G. and Tawn, J. A. (1990) Statistics of coastal flood prevention. *Phil. Trans. R. Soc. Lond., A* 332, 457–476.

Description

The sealevel2 data frame has 81 rows and 3 columns. The first two columns contain annual sea level maxima from 1912 to 1992 at Dover and Harwich respectively, two sites on the coast of Britain. The third column is a logical vector denoting whether or not the maxima in a given year are assumed to have derived from the same storm event; this assumption is made if the times of obsevation of the maxima are at most 48 hours apart. The row names give the years of observation. There are 39 missing data values. There are only nine non-missing logical values.

Usage

sealevel2

Format

This data frame contains the following columns:

dover A numeric vector containing annual sea level maxima at Dover, including 9 missing values.

- harwich A numeric vector containing sea annual level maxima at Harwich, including 30 missing values.
- case A logical vector denoting whether or not the maxima are assumed to have derived from the same storm event.

Source

Coles, S. G. and Tawn, J. A. (1990) Statistics of coastal flood prevention. *Phil. Trans. R. Soc. Lond., A* 332, 457–476.

tcplot *Threshold Choice Plot*

Description

Plots of parameter estimates at various thresholds for peaks over threshold modelling, using the Generalized Pareto or Point Process representation.

Usage

```
tcplot(data, tlim, model = c("gpd", "pp"), pscale = FALSE, cmax =FALSE, r = 1, ulow = -Inf, rlow = 1, nt = 25, which = 1:npar,
   conf = 0.95, lty = 1, lwd = 1, type = "b", cilty = 1, vci =
   TRUE, xlab, xlim, ylabs, ylims, ask = nb.fig < length(which) &&
   dev.interactive(), ...)
```
tcplot 85

Arguments

Details

For each of the nt thresholds a peaks over threshold model is fitted using the function fpot. When model = "gpd" (the default), the maximum likelihood estimates for the shape and the modified scale parameter (modified by subtracting the shape multiplied by the threshold) are plotted against the thresholds. When model = "pp" the maximum likelihood estimates for the location, scale and shape parameters are plotted against the thresholds. (The modified scale parameter in the "gpd" case is equivalent to the scale parameter in the "pp" case.) If the threshold u is a valid threshold to be used for peaks over threshold modelling, the parameter estimates depicted should be approximately constant above u.

Value

A list is invisibly returned. Each component is a matrix with three columns giving parameter estimates and confidence limits.

Author(s)

Stuart Coles and Alec Stephenson

See Also

[fpot](#page-46-0), [mrlplot](#page-59-0), [clusters](#page-21-0)

Examples

```
tlim \leq c(3.6, 4.2)## Not run: tcplot(portpirie, tlim)
## Not run: tcplot(portpirie, tlim, nt = 100, lwd = 3, type = "l")
## Not run: tcplot(portpirie, tlim, model = "pp")
```
uccle *Rainfall Maxima at Uccle, Belgium*

Description

The uccle data frame has 35 rows and 4 columns. The columns contain annual rainfall maxima (in millimetres) from 1938 to 1972 at Uccle, Belgium, over the durations of one day, one hour, ten minutes and one minute. The row names give the years of observation.

Usage

uccle

Format

This data frame contains the following columns:

day Annual daily rainfall maxima.

hour Annual hourly rainfall maxima.

tmin Annual rainfall maxima over ten minute durations.

min Annual rainfall maxima over one minute durations.

Source

Sneyers, R. (1977) L'intensite maximale des precipitations en Belgique. *Inst. Royal Meteor. Belgique, B* 86.

Description

The venice data frame has 51 rows and 10 columns. The jth column contains the jth largest sea levels in Venice, for the years 1931–1981. Only the largest six measurements are available for the year 1935; the corresponding row contains four missing values. The years for each set of measurements are given as row names. A larger version of this data is available in the dataset venice2.

Usage

venice

Format

A data frame with 51 rows and 10 columns.

Source

Smith, R. L. (1986) Extreme value theory based on the r largest annual events. *Journal of Hydrology*, 86, 27–43.

References

Coles, S. G. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

venice2 *Largest Sea Levels in Venice*

Description

The venice2 data frame has 125 rows and 10 columns. The data was kindly provided by Anthony Davison. The jth column contains the jth largest sea levels in Venice, for the years 1887–2011. This is a larger version of the dataset venice. Only the largest six measurements are available for the year 1935, and only the largest is available for 1922; the corresponding rows contain missing values. The years for each set of measurements are given as row names.

Usage

venice2

Format

A data frame with 125 rows and 10 columns.

Source

Smith, R. L. (1986) Extreme value theory based on the r largest annual events. *Journal of Hydrology*, 86, 27–43.

References

Coles, S. G. (2001) *An Introduction to Statistical Modeling of Extreme Values*. London: Springer-Verlag.

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