

Fitting genotype by environment models in sommer

Giovanny Covarrubias-Pazaran

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The sommer package was developed to provide R users a powerful and reliable multivariate mixed model solver. The package is focused on two approaches: 1) $p > n$ (more effects to estimate than observations) using the `mmer()` function, and 2) $n > p$ (more observations than effects to estimate) using the `mmec()` function. The core algorithms are coded in C++ using the Armadillo library. This package allows the user to fit mixed models with the advantage of specifying the variance-covariance structure for the random effects, specifying heterogeneous variances, and obtaining other parameters such as BLUPs, BLUEs, residuals, fitted values, variances for fixed and random effects, etc.

The purpose of this vignette is to show how to fit different genotype by environment (GxE) models using the sommer package:

- 1) Single environment model
- 2) Multienvironment model: Main effect model
- 3) Multienvironment model: Diagonal model (DG)
- 4) Multienvironment model: Compound symmetry model (CS)
- 5) Multienvironment model: Unstructured model (US)
- 6) Multienvironment model: Random regression model (RR)
- 7) Multienvironment model: Other covariance structures for GxE
- 8) Multienvironment model: Finlay-Wilkinson regression
- 9) Multienvironment model: Factor analytic (reduced rank) model (FA)
- 10) Two stage analysis

When the breeder decides to run a trial and apply selection in a single environment (whether because the amount of seed is a limitation or there's no availability for a location) the breeder takes the risk of selecting material for a target population of environments (TPEs) using an environment that is not representative of the larger TPE. Therefore, many breeding programs try to base their selection decision on multi-environment trial (MET) data. Models could be adjusted by adding additional information like spatial information, experimental design information, etc. In this tutorial we will focus mainly on the covariance structures for GxE and the incorporation of relationship matrices for the genotype effect.

1) Single environment model

A single-environment model is the one that is fitted when the breeding program can only afford one location, leaving out the possible information available from other environments. This will be used to further expand to GxE models.

```
library(sommer)
data(DT_example)
DT <- DT_example
A <- A_example

ansSingle <- mmer(Yield~1,
                 random= ~ vsr(Name, Gu=A),
```

```

rcov= ~ units,
data=DT, verbose = FALSE)
summary(ansSingle)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -78.80875 159.6175 162.8378      NR      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield  6.529      2.202  2.965  Positive
## units.Yield-Yield  13.868      1.633  8.494  Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept)  11.74      0.4876  24.07
## =====
## Groups and observations:
##   Yield
## u:Name  41
## =====
## Use the '$' sign to access results and parameters

```

```

# or
Ai <- as(solve(A), Class="dgCMatrix")
ansSingle <- mmec(Yield~1,
  random= ~ vsc(isc(Name), Gu=Ai),
  rcov= ~ units,
  data=DT, verbose = FALSE)
summary(ansSingle)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -359.0031 720.0062 723.2266      AI      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## Name:Ai:isc:isc  6.495      1.479  4.392  Positive
## units:isc:isc  13.869      1.799  7.711  Positive
## =====
## Fixed effects:
##           Estimate Std.Error t.value
## (Intercept)  11.74      0.4861  24.15
## =====
## Use the '$' sign to access results and parameters

```

In this model, the only term to be estimated is the one for the germplasm (here called Name). For the sake of example we have added a relationship matrix among the levels of the random effect Name. This is just a diagonal matrix with as many rows and columns as levels present in the random effect Name, but any other

non-diagonal relationship matrix could be used.

2) MET: main effect model

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The main effect model assumes that GxE doesn't exist and that the main genotype effect plus the fixed effect for environment is enough to predict the genotype effect in all locations of interest.

```
ansMain <- mmer(Yield~Env,
               random= ~ vsr(Name, Gu=A),
               rcov= ~ units,
               data=DT, verbose = FALSE)
summary(ansMain)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -32.59421 71.18842 80.84949      NR      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield  4.856    1.5233  3.188  Positive
## units.Yield-Yield  8.109    0.9615  8.434  Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept)  16.385    0.5849  28.012
## 2 Yield  EnvCA.2012   -5.688    0.5741  -9.908
## 3 Yield  EnvCA.2013   -6.218    0.6107 -10.182
## =====
## Groups and observations:
##           Yield
## u:Name    41
## =====
## Use the '$' sign to access results and parameters
```

```
# or

Ai <- as(solve(A), Class="dgCMatrix")
ansMain <- mmec(Yield~Env,
               random= ~ vsc(isc(Name), Gu=Ai),
               rcov= ~ units,
               data=DT, verbose = FALSE)
summary(ansMain)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -313.3005 632.6011 642.2621      AI      TRUE
## =====
## Variance-Covariance components:
```

```
##                               VarComp VarCompSE Zratio Constraint
## Name:Ai:isc:isc      4.854      1.449  3.350   Positive
## units:isc:isc       8.109      1.807  4.486   Positive
## =====
## Fixed effects:
##           Estimate Std.Error t.value
## Intercept  16.385    0.5847  28.021
## CA.2012    -5.688    0.5741  -9.909
## CA.2013    -6.219    0.6107 -10.183
## =====
## Use the '$' sign to access results and parameters
```

3) MET: diagonal model (DG)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The diagonal model assumes that GxE exists and that the genotype variation is expressed differently at each location, therefore fitting a variance component for the genotype effect at each location. The main drawback is that this model assumes no covariance among locations, as if genotypes were independent (despite the fact that is the same genotypes). The fixed effect for environment plus the location-specific BLUP is used to predict the genotype effect in each locations of interest.

```
ansDG <- mmer(Yield~Env,
              random= ~ vsr(dsr(Env),Name, Gu=A),
              rcov= ~ units,
              data=DT, verbose = FALSE)
summary(ansDG)
```

```
## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -21.04157 48.08315 57.74421      NR      TRUE
## =====
## Variance-Covariance components:
##                               VarComp VarCompSE Zratio Constraint
## CA.2011:Name.Yield-Yield  17.493    6.1099  2.863   Positive
## CA.2012:Name.Yield-Yield   5.337    1.7662  3.022   Positive
## CA.2013:Name.Yield-Yield   7.884    2.5526  3.089   Positive
## units.Yield-Yield          4.381    0.6493  6.747   Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept)  16.621    0.948  17.532
## 2 Yield EnvCA.2012  -5.958    1.045  -5.699
## 3 Yield EnvCA.2013  -6.662    1.098  -6.067
## =====
## Groups and observations:
##           Yield
## CA.2011:Name    41
## CA.2012:Name    41
## CA.2013:Name    41
## =====
## Use the '$' sign to access results and parameters
```

```

# or
Ai <- as(solve(A), Class="dgCMatrix")
ansDG <- mmec(Yield~Env,
             random= ~ vsc(dsc(Env),isc(Name), Gu=Ai),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansDG)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -301.7528 609.5057 619.1668      AI      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## Env:Name:AI:CA.2011:CA.2011  17.124    3.479  4.922  Positive
## Env:Name:AI:CA.2012:CA.2012   5.351    2.814  1.901  Positive
## Env:Name:AI:CA.2013:CA.2013   7.904    2.924  2.703  Positive
## units:isc:isc                4.332    2.258  1.919  Positive
## =====
## Fixed effects:
##           Estimate Std.Error t.value
## Intercept    16.621    0.9386  17.709
## CA.2012      -5.958    1.0367  -5.747
## CA.2013      -6.662    1.0898  -6.113
## =====
## Use the '$' sign to access results and parameters

```

4) MET: compound symmetry model (CS)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The compound symmetry model assumes that GxE exists and that a main genotype variance-covariance component is expressed across all location. In addition, it assumes that a main genotype-by-environment variance is expressed across all locations. The main drawback is that the model assumes the same variance and covariance among locations. The fixed effect for environment plus the main effect for BLUP plus genotype-by-environment effect is used to predict the genotype effect in each location of interest.

```

E <- diag(length(unique(DT$Env)))
rownames(E) <- colnames(E) <- unique(DT$Env)
EA <- kronecker(E,A, make.dimnames = TRUE)
ansCS <- mmer(Yield~Env,
             random= ~ vsr(Name, Gu=A) + vsr(Env:Name, Gu=EA),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansCS)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -20.14538 46.29075 55.95182      NR      TRUE

```

```

## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield      3.682      1.691  2.177  Positive
## u:Env:Name.Yield-Yield  5.173      1.495  3.460  Positive
## units.Yield-Yield      4.366      0.647  6.748  Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept)  16.496      0.6855  24.065
## 2 Yield  EnvCA.2012   -5.777      0.7558  -7.643
## 3 Yield  EnvCA.2013   -6.380      0.7960  -8.015
## =====
## Groups and observations:
##           Yield
## u:Name      41
## u:Env:Name  123
## =====
## Use the '$' sign to access results and parameters

## or
E <- diag(length(unique(DT$Env)));rownames(E) <- colnames(E) <- unique(DT$Env)
Ei <- solve(E)
Ai <- solve(A)
EAi <- kronecker(Ei,Ai, make.dimnames = TRUE)
Ei <- as(Ei, Class="dgCMatrix")
Ai <- as(Ai, Class="dgCMatrix")
EAi <- as(EAi, Class="dgCMatrix")
ansCS <- mmec(Yield~Env,
              random= ~ vsc(isc(Name), Gu=Ai) + vsc(isc(Env:Name), Gu=EAi),
              rcov= ~ units,
              data=DT, verbose = FALSE)
summary(ansCS)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -300.8517 607.7034 617.3645      AI      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## Name:Ai:isc:isc      3.683      1.881  1.958  Positive
## Env:Name:EAi:isc:isc  5.174      2.420  2.138  Positive
## units:isc:isc      4.360      2.270  1.920  Positive
## =====
## Fixed effects:
##           Estimate Std.Error t.value
## Intercept  16.496      0.6856  24.062
## CA.2012    -5.777      0.7558  -7.643
## CA.2013    -6.380      0.7960  -8.015
## =====
## Use the '$' sign to access results and parameters

```

5) MET: unstructured model (US)

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The unstructured model is the most flexible model assuming that GxE exists and that an environment-specific variance exists in addition to as many covariances for each environment-to-environment combinations. The main drawback is that is difficult to make this models converge because of the large number of variance components, the fact that some of these variance or covariance components are zero, and the difficulty in choosing good starting values. The fixed effect for environment plus the environment specific BLUP (adjusted by covariances) is used to predict the genotype effect in each location of interest.

```
ansUS <- mmer(Yield~Env,
             random= ~ vsr(usr(Env),Name, Gu=A),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansUS)

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -14.20951 34.41901 44.08008      NR      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## CA.2011:Name.Yield-Yield      15.994      5.381 2.972 Positive
## CA.2012:CA.2011:Name.Yield-Yield 6.172      2.503 2.465 Unconstr
## CA.2012:Name.Yield-Yield      5.273      1.750 3.013 Positive
## CA.2013:CA.2011:Name.Yield-Yield 6.366      3.069 2.074 Unconstr
## CA.2013:CA.2012:Name.Yield-Yield 0.376      1.535 0.245 Unconstr
## CA.2013:Name.Yield-Yield      7.689      2.490 3.088 Positive
## units.Yield-Yield      4.386      0.650 6.748 Positive
## =====
## Fixed effects:
## Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.341      0.8141 20.072
## 2 Yield EnvCA.2012 -5.696      0.7406 -7.692
## 3 Yield EnvCA.2013 -6.286      0.8202 -7.664
## =====
## Groups and observations:
##           Yield
## CA.2011:Name      41
## CA.2012:CA.2011:Name 82
## CA.2012:Name      41
## CA.2013:CA.2011:Name 82
## CA.2013:CA.2012:Name 82
## CA.2013:Name      41
## =====
## Use the '$' sign to access results and parameters

# adjust variance BLUPs by adding covariances
# ansUS$U[1:6] <- unsBLUP(ansUS$U[1:6])

# or
Ai <- solve(A)
```

```

Ai <- as(Ai, Class="dgCMatrix")
ansUS <- mmec(Yield~Env,
             random= ~ vsc(usc(Env),isc(Name), Gu=Ai),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansUS)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -295.2125 596.4249 606.086      AI      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## Env:Name:AI:CA.2011:CA.2011 14.8143      3.513 4.2167 Positive
## Env:Name:AI:CA.2011:CA.2012  5.7221      1.957 2.9241 Unconstr
## Env:Name:AI:CA.2012:CA.2012  4.9599      2.263 2.1913 Positive
## Env:Name:AI:CA.2011:CA.2013  6.1289      2.726 2.2481 Unconstr
## Env:Name:AI:CA.2012:CA.2013  0.6439      1.899 0.3391 Unconstr
## Env:Name:AI:CA.2013:CA.2013  7.5772      2.905 2.6084 Positive
## units:isc:isc                4.0134      2.171 1.8483 Positive
## =====
## Fixed effects:
##           Estimate Std.Error t.value
## Intercept  16.344    0.7847 20.828
## CA.2012    -5.693    0.7150 -7.963
## CA.2013    -6.277    0.7918 -7.927
## =====
## Use the '$' sign to access results and parameters

```

6) MET: random regression model

A multi-environment model is the one that is fitted when the breeding program can afford more than one location. The random regression model assumes that the environment can be seen as a continuous variable and therefore a variance component for the intercept and a variance component for the slope can be fitted. The number of variance components will depend on the order of the Legendre polynomial fitted.

```

library(orthopolynom)
DT$EnvN <- as.numeric(as.factor(DT$Env))
ansRR <- mmer(Yield~Env,
             random= ~ vsr(leg(EnvN,1),Name),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansRR)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -27.70318 61.40636 71.06743      NR      TRUE
## =====

```



```
## Variance-Covariance components:
##              VarComp VarCompSE Zratio Constraint
## leg0:Name.Yield-Yield 10.392   3.1473  3.302   Positive
## leg1:Name.Yield-Yield  2.079   0.9792  2.123   Positive
## units.Yield-Yield     6.297   0.8442  7.459   Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.541    0.6770 24.432
## 2 Yield EnvCA.2012  -5.832    0.6425 -9.078
## 3 Yield EnvCA.2013  -6.472    0.8239 -7.854
## =====
## Groups and observations:
##      Yield
## leg0:Name  41
## leg1:Name  41
## =====
## Use the '$' sign to access results and parameters
```

```
# or

ansRR <- mmec(Yield~Env,
              random= ~ vsc(dsc(leg(EnvN,1)),isc(Name)),
              rcov= ~ units,
              data=DT, verbose = FALSE)
summary(ansRR)
```

```
## =====
##          Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##          logLik      AIC      BIC Method Converge
## Value -308.4098 622.8195 632.4806      AI      TRUE
## =====
## Variance-Covariance components:
##              VarComp VarCompSE Zratio Constraint
## EnvN:Name:leg0:leg0 10.355    2.319  4.465   Positive
## EnvN:Name:leg1:leg1  2.079    1.759  1.182   Positive
## units:isc:isc       6.304    2.005  3.145   Positive
## =====
## Fixed effects:
##          Estimate Std.Error t.value
## Intercept 16.541    0.6761 24.467
## CA.2012   -5.833    0.6421 -9.084
## CA.2013   -6.472    0.8233 -7.861
## =====
## Use the '$' sign to access results and parameters
```

In addition, an unstructured, diagonal or other variance-covariance structure can be put on top of the polynomial model:

```
library(orthopolynom)
DT$EnvN <- as.numeric(as.factor(DT$Env))
ansRR <- mmer(Yield~Env,
              random= ~ vsr(usr(leg(EnvN,1)),Name),
              rcov= ~ units,
```

```

data=DT, verbose = FALSE)
summary(ansRR)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -25.56967 57.13935 66.80042      NR      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## leg0:Name.Yield-Yield      10.791      3.2745 3.295 Positive
## leg1:leg0:Name.Yield-Yield -2.428      1.3699 -1.772 Unconstr
## leg1:Name.Yield-Yield      2.286      1.0404 2.197 Positive
## units.Yield-Yield      6.260      0.8421 7.434 Positive
## =====
## Fixed effects:
## Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept) 16.501      0.7778 21.216
## 2 Yield EnvCA.2012 -5.791      0.6704 -8.638
## 3 Yield EnvCA.2013 -6.476      0.8554 -7.570
## =====
## Groups and observations:
##           Yield
## leg0:Name      41
## leg1:leg0:Name 82
## leg1:Name      41
## =====
## Use the '$' sign to access results and parameters

```

```
# or
```

```

ansRR <- mmec(Yield~Env,
              random= ~ vsc(usc(leg(EnvN,1)),isc(Name)),
              rcov= ~ units,
              data=DT, verbose = FALSE)
summary(ansRR)

```

```

## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -308.7042 623.4085 633.0695      AI      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## EnvN:Name:leg0:leg0 10.6512      2.242 4.75048 Positive
## EnvN:Name:leg0:leg1 0.1204      1.445 0.08328 Unconstr
## EnvN:Name:leg1:leg1 2.0947      1.920 1.09114 Positive
## units:isc:isc      6.6403      2.042 3.25168 Positive
## =====
## Fixed effects:

```

```
##           Estimate Std.Error t.value
## Intercept   16.541    0.6826  24.234
## CA.2012     -5.833    0.6533  -8.929
## CA.2013     -6.469    0.8349  -7.748
## =====
## Use the '$' sign to access results and parameters
```

7) Other GxE covariance structures

Although not very commonly used in GxE models, the autoregressive of order 1 (AR1) and other covariance structures could be used in the GxE modeling. Here we show how to do it (not recommending it).

```
E <- AR1(DT$Env) # can be AR1() or CS(), etc.
rownames(E) <- colnames(E) <- unique(DT$Env)
EA <- kronecker(E,A, make.dimnames = TRUE)
ansCS <- mmer(Yield~Env,
             random= ~ vsr(Name, Gu=A) + vsr(Env:Name, Gu=EA),
             rcov= ~ units,
             data=DT, verbose = FALSE)
summary(ansCS)
```

```
## =====
##           Multivariate Linear Mixed Model fit by REML
## ***** sommer 4.3 *****
## =====
##           logLik      AIC      BIC Method Converge
## Value -19.39067 44.78134 54.4424      NR      TRUE
## =====
## Variance-Covariance components:
##           VarComp VarCompSE Zratio Constraint
## u:Name.Yield-Yield      2.225   1.7536  1.269   Positive
## u:Env:Name.Yield-Yield  6.424   1.8293  3.512   Positive
## units.Yield-Yield      4.334   0.6418  6.752   Positive
## =====
## Fixed effects:
##   Trait      Effect Estimate Std.Error t.value
## 1 Yield (Intercept)  16.484    0.6735  24.474
## 2 Yield  EnvCA.2012  -5.780    0.7365  -7.848
## 3 Yield  EnvCA.2013  -6.372    0.7799  -8.170
## =====
## Groups and observations:
##           Yield
## u:Name      41
## u:Env:Name  123
## =====
## Use the '$' sign to access results and parameters
```

8) Finlay-Wilkinson regression

```
data(DT_h2)
DT <- DT_h2
```

```

## build the environmental index
ei <- aggregate(y~Env, data=DT,FUN=mean)
colnames(ei)[2] <- "envIndex"
ei$envIndex <- ei$envIndex - mean(ei$envIndex,na.rm=TRUE) # center the envIndex to have clean VCs
ei <- ei[with(ei, order(envIndex)), ]

## add the environmental index to the original dataset
DT2 <- merge(DT,ei, by="Env")

# numeric by factor variables like envIndex:Name can't be used in the random part like this
# they need to come with the vsc() structure
DT2 <- DT2[with(DT2, order(Name)), ]
mix2 <- mmec(y~ envIndex,
             random=~ Name + vsc(dsc(envIndex),isc(Name)), data=DT2,
             rcov=~vsc(dsc(Name),isc(units)),
             tolParConvNorm = .0001,
             nIters = 50, verbose = FALSE
            )

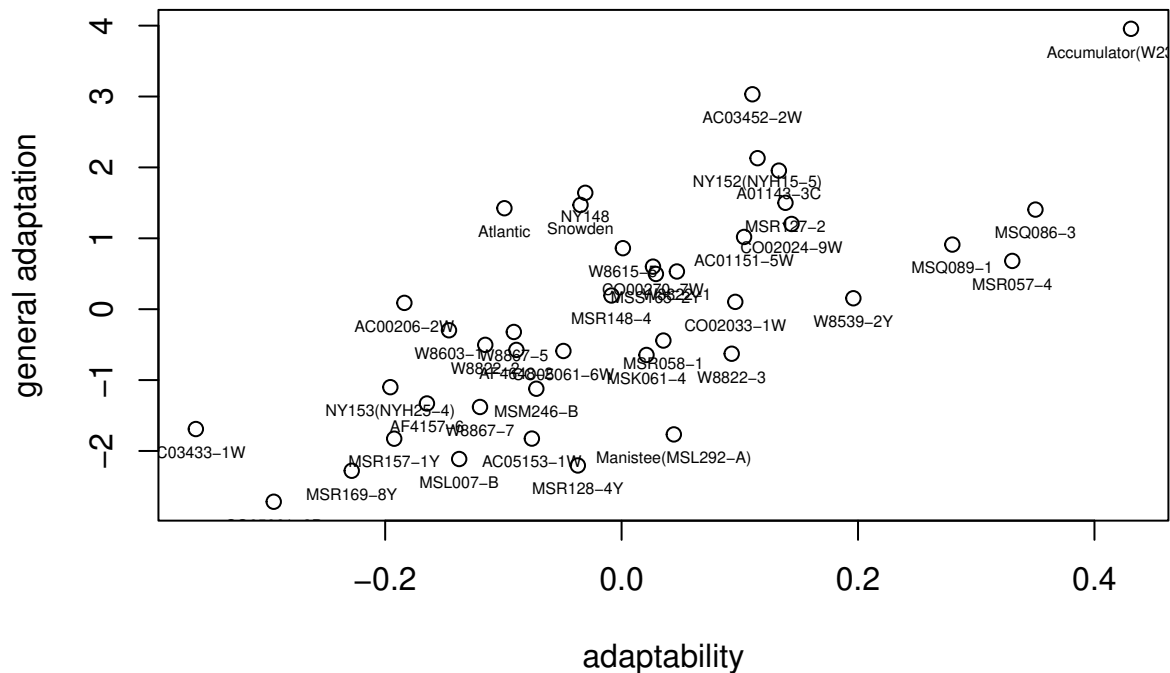
## 'as(<dgeMatrix>, "dgCMatrix")' is deprecated.
## Use 'as(., "CsparseMatrix")' instead.
## See help("Deprecated") and help("Matrix-deprecated").

# summary(mix2)$varcomp

b=mix2$uList$vsc(dsc(envIndex), isc(Name))` # adaptability (b) or genotype slopes
mu=mix2$uList$vsc(isc(Name))` # general adaptation (mu) or main effect
e=sqrt(summary(mix2)$varcomp[-c(1:2),1]) # error variance for each individual

## general adaptation (main effect) vs adaptability (response to better environments)
plot(mu[,1]~b[,1], ylab="general adaptation", xlab="adaptability")
text(y=mu[,1],x=b[,1], labels = rownames(mu), cex=0.5, pos = 1)

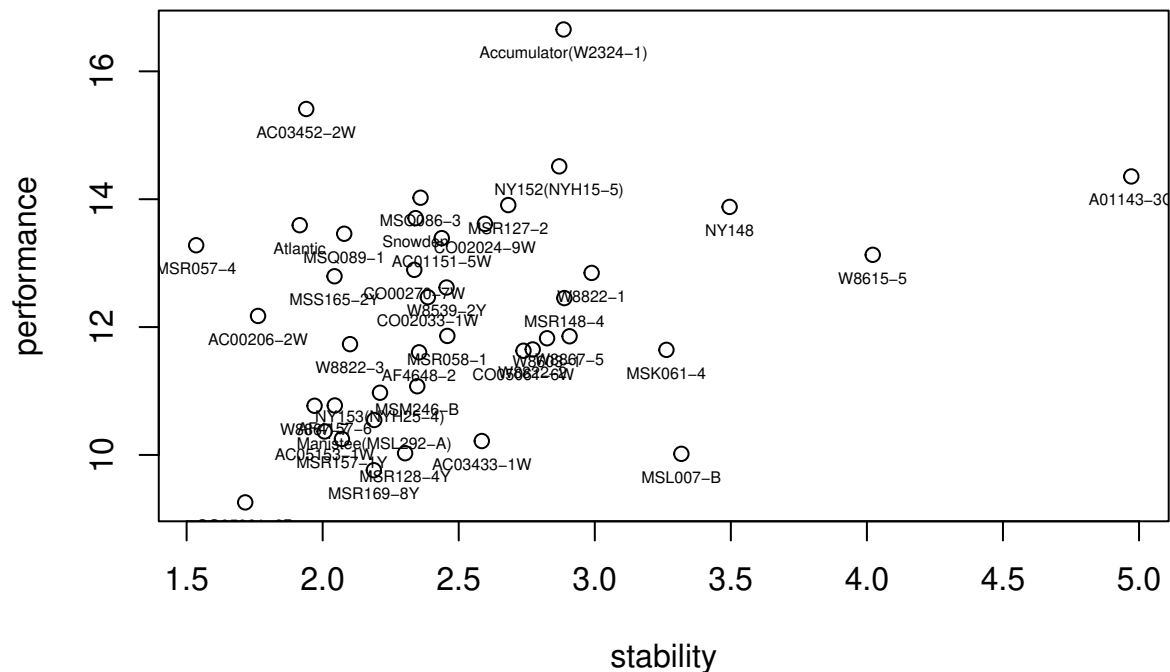
```



```

## prediction across environments
Dt <- mix2$Dtable
Dt[1,"average"]=TRUE
Dt[2,"include"]=TRUE
Dt[3,"include"]=TRUE
pp <- predict(mix2,Dtable = Dt, D="Name")
preds <- pp$pvals
# preds[with(preds, order(-predicted.value)), ]
## performance vs stability (deviation from regression line)
plot(preds[,2]~e, ylab="performance", xlab="stability")
text(y=preds[,2],x=e, labels = rownames(mu), cex=0.5, pos = 1)

```



9) Factor analytic (reduced rank) model

When the number of environments where genotypes are evaluated is big and we want to consider the genetic covariance between environments and location-specific variance components we cannot fit an unstructured covariance in the model since the number of parameters is too big and the matrix can become non-full rank leading to singularities. In those cases is suggested a dimensionality reduction technique. Among those the factor analytic structures proposed by many research groups (Piepho, Smith, Cullis, Thompson, Meyer, etc.) are the way to go. Sommer has a reduced-rank factor analytic implementation available through the `rrc()` function. Here we show an example of how to fit the model:

```

data(DT_h2)
DT <- DT_h2
DT=DT[with(DT, order(Env)), ]
head(DT)

```

```

##      Name      Env Loc Year   Block y
## 67  MSL007-B CA.2011 CA 2011 CA.2011.2 5
## 105 MSL007-B CA.2011 CA 2011 CA.2011.1 6
## 308 MSK061-4 CA.2011 CA 2011 CA.2011.2 9
## 393 MSK061-4 CA.2011 CA 2011 CA.2011.1 10

```

```

## 469 MSR169-8Y CA.2011 CA 2011 CA.2011.1 11
## 471 NY148 CA.2011 CA 2011 CA.2011.1 11

indNames <- na.omit(unique(DT$Name))
A <- diag(length(indNames))
rownames(A) <- colnames(A) <- indNames

# fit diagonal model first to produce H matrix
ansDG <- mmec(y~Env,
              random=~ vsc(dsc(Env), isc(Name)),
              rcov=~units, nIters = 100,
              # we recommend giving more EM iterations at the beginning
              emWeight = c(rep(1,10),logspace(10,1,.05), rep(.05,80)),
              data=DT, verbose = FALSE)

HO <- ansDG$uList$`vsc(dsc(Env), isc(Name))` # GxE table

# reduced rank model
ansFA <- mmec(y~Env,
              random=~vsc( usc(rrc(Env, H = HO, nPC = 3)) , isc(Name)) + # rr
                vsc(dsc(Env), isc(Name)), # diag
              rcov=~units,
              # we recommend giving more iterations to these models
              nIters = 100, verbose = FALSE,
              # we recommend giving more EM iterations at the beginning
              emWeight = c(rep(1,10),logspace(10,1,.05), rep(.05,80)),
              data=DT)

vcFA <- ansFA$theta[[1]]
vcDG <- ansFA$theta[[2]]

loadings=with(DT, rrc(Env, nPC = 3, H = HO, returnGamma = TRUE) )$Gamma
scores <- ansFA$uList[[1]]

vcUS <- loadings %*% vcFA %*% t(loadings)
G <- vcUS + vcDG
# colfunc <- colorRampPalette(c("steelblue4","springgreen","yellow"))
# hv <- heatmap(cov2cor(G), col = colfunc(100), symm = TRUE)

uFA <- scores %*% t(loadings)
uDG <- ansFA$uList[[2]]
u <- uFA + uDG

```

As can be seen genotype BLUPs for all environments can be recovered by multiplying the loadings (Gamma) by the factor scores. This is a parsimonious way to model an unstructured covariance.

10) Two stage analysis

It is common then to fit a first model that accounts for the variation of random design elements, e.g., locations, years, blocks, and fixed genotype effects to obtain the estimated marginal means (EMMs) or best linear unbiased estimators (BLUES) as adjusted entry means. These adjusted entry means are then used as the phenotype or response variable in GWAS and genomic prediction studies.

```
#####
## stage 1
## use mmer for dense field trials
#####
```

```
data(DT_h2)
DT <- DT_h2
head(DT)
```

```
##           Name      Env Loc Year      Block y
## 1           W8822-3 FL.2012  FL 2012  FL.2012.1 2
## 2           W8867-7 FL.2012  FL 2012  FL.2012.2 2
## 3           MSL007-B MO.2011  MO 2011  MO.2011.1 3
## 4           C000270-7W FL.2012  FL 2012  FL.2012.2 3
## 5 Manistee(MSL292-A) FL.2013  FL 2013  FL.2013.2 3
## 6           MSM246-B FL.2012  FL 2012  FL.2012.2 3
```

```
envs <- unique(DT$Env)
BLUEL <- list()
XtXL <- list()
for(i in 1:length(envs)){
  ans1 <- mmer(y~Name-1,
              random=~Block,
              verbose=FALSE,
              data=droplevels(DT[which(DT$Env == envs[i]),]
              )
  )
  ans1$Beta$Env <- envs[i]

  BLUEL[[i]] <- ans1$Beta
  # to be comparable to 1/(se^2) = 1/PEV = 1/Ci = 1/[(X'X)inv]
  XtXL[[i]] <- solve(ans1$VarBeta)
}

```

```
DT2 <- do.call(rbind, BLUEL)
OM <- do.call(adiag1,XtXL)
```

```
#####
## stage 2
## use mmec for sparse equation
#####
```

```
m <- matrix(1/var(DT2$Estimate, na.rm = TRUE))
ans2 <- mmec(Estimate~Env,
             random=~Effect + Env:Effect,
             rcov=~vsc(isc(units,thetaC = matrix(3), theta = m)),
             W=OM,
             verbose=FALSE,
             data=DT2
             )
```

```
## Using the weights matrix
```

```
summary(ans2)$varcomp
```

```
##           VarComp VarCompSE      Zratio Constraint
## Effect:isc:isc    2.076920 0.5758219  3.606880   Positive
```

```
## Env:Effect:isc:isc 3.333792 0.1850479 18.015831 Positive
## units:m:          1.000000 0.1850479 5.404005 Fixed
```

Literature

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