# Package 'strucchangeRcpp'

October 17, 2024

Title Testing, Monitoring, and Dating Structural Changes: C++ Version
 Description A fast implementation with additional experimental features for testing, monitoring and dating structural changes in (linear) regression models. 'strucchangeRcpp' features tests/methods from the generalized fluctuation test framework as well as from

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Friedrich Leisch [aut],

the F test (Chow test) framework. This includes methods to fit, plot and test fluctuation processes (e.g. cumulative/moving sum, recursive/moving estimates) and F statistics, respectively. These methods are described in Zeileis et al. (2002) <doi:10.18637/jss.v007.i02>. Finally, the breakpoints in regression models with structural changes can be estimated together with confidence intervals, and their magnitude as well as the model fit can be evaluated using a variety of statistical measures. LazyData yes LinkingTo Rcpp, RcppArmadillo **Depends** R (>= 2.10.0), zoo, sandwich **Suggests** stats4, car, dynlm, e1071, foreach, lmtest, mytnorm, tseries, bfast **Imports** graphics, stats, Rcpp (>= 0.12.7), utils License GPL-2 | GPL-3 URL https://github.com/bfast2/strucchangeRcpp/ BugReports https://github.com/bfast2/strucchangeRcpp/issues RoxygenNote 7.1.1 **Encoding UTF-8 NeedsCompilation** yes Author Dainius Masiliunas [aut, cre] (<a href="https://orcid.org/0000-0001-5654-1277">https://orcid.org/0000-0001-5654-1277</a>), Achim Zeileis [aut] (<a href="https://orcid.org/0000-0003-0918-3766">https://orcid.org/0000-0003-0918-3766</a>), Marius Appel [aut],

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# Description

Data about the number of youth homicides in Boston during the 'Boston Gun Project'—a policing initiative aiming at lowering homicide victimization among young people in Boston.

### Usage

```
data("BostonHomicide")
```

#### **Format**

A data frame containing 6 monthly time series and two factors coding seasonality and year, respectively.

homicides time series. Number of youth homicides.

population time series. Boston population (aged 25-44), linearly interpolated from annual data.

**populationBM** time series. Population of black males (aged 15-24), linearly interpolated from annual data.

ahomicides25 time series. Number of adult homicides (aged 25 and older).

ahomicides35 time series. Number of adult homicides (aged 35-44).

**unemploy** time series. Teen unemployment rate (in percent).

season factor coding the month.

year factor coding the year.

### **Details**

The 'Boston Gun Project' is a policing initiative aiming at lowering youth homicides in Boston. The project began in early 1995 and implemented the so-called 'Operation Ceasefire' intervention which began in the late spring of 1996.

### Source

Piehl et al. (2004), Figure 1, Figure 3, and Table 1.

From the table it is not clear how the data should be linearly interpolated. Here, it was chosen to use the given observations for July of the corresponding year and then use approx with rule = 2.

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### References

Piehl A.M., Cooper S.J., Braga A.A., Kennedy D.M. (2003), Testing for Structural Breaks in the Evaluation of Programs, *The Review of Economics and Statistics*, **85**(3), 550-558.

Kennedy D.M., Piehl A.M., Braga A.A. (1996), Youth Violence in Boston: Gun Markets, Serious Youth Offenders, and a Use-Reduction Strategy, *Law and Contemporary Problems*, **59**, 147-183.

### **Examples**

```
data("BostonHomicide")
attach(BostonHomicide)
## data from Table 1
tapply(homicides, year, mean)
populationBM[0:6*12 + 7]
tapply(ahomicides25, year, mean)
tapply(ahomicides35, year, mean)
population[0:6*12 + 7]
unemploy[0:6*12 + 7]
## model A
## via OLS
fmA <- lm(homicides ~ populationBM + season)</pre>
anova(fmA)
## as GLM
fmA1 <- glm(homicides ~ populationBM + season, family = poisson)</pre>
anova(fmA1, test = "Chisq")
## model B & C
fmB <- lm(homicides ~ populationBM + season + ahomicides25)</pre>
fmC <- lm(homicides ~ populationBM + season + ahomicides25 + unemploy)</pre>
detach(BostonHomicide)
```

boundary

Boundary Function for Structural Change Tests

#### **Description**

A generic function computing boundaries for structural change tests

#### Usage

```
boundary(x, ...)
```

### Arguments

x an object. Use methods to see which class has a method for boundary.

... additional arguments affecting the boundary.

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# Value

an object of class "ts" with the same time properties as the time series in x

# See Also

```
boundary.efp, boundary.Fstats
```

boundary.efp

Boundary for Empirical Fluctuation Processes

# Description

Computes boundary for an object of class "efp"

# Usage

```
## S3 method for class 'efp'
boundary(x, alpha = 0.05, alt.boundary = FALSE,
  functional = "max", ...)
```

### **Arguments**

x	an object of class "efp".
alpha	numeric from interval $(0,1)$ indicating the confidence level for which the boundary of the corresponding test will be computed.
alt.boundary	logical. If set to TRUE alternative boundaries (instead of the standard linear boundaries) will be computed (for Brownian bridge type processes only).
functional	indicates which functional should be applied to the empirical fluctuation process. See also plot.efp.
• • •	currently not used.

### Value

an object of class "ts" with the same time properties as the process in x

### See Also

```
efp, plot.efp
```

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### **Examples**

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data("nhtemp")
## plot the data
plot(nhtemp)

## test the model null hypothesis that the average temperature remains constant
## over the years
## compute OLS-CUSUM fluctuation process
temp.cus <- efp(nhtemp ~ 1, type = "OLS-CUSUM")
## plot the process without boundaries
plot(temp.cus, alpha = 0.01, boundary = FALSE)
## add the boundaries in another colour
bound <- boundary(temp.cus, alpha = 0.01)
lines(bound, col=4)
lines(-bound, col=4)</pre>
```

boundary.Fstats

Boundary for F Statistics

### **Description**

Computes boundary for an object of class "Fstats"

# Usage

### **Arguments**

X	an object of class "Fstats".
alpha	numeric from interval $(0,1)$ indicating the confidence level for which the boundary of the supF test will be computed.
pval	logical. If set to TRUE a boundary for the corresponding ${\bf p}$ values will be computed.
aveF	logical. If set to TRUE the boundary of the aveF (instead of the supF) test will be computed. The resulting boundary then is a boundary for the mean of the F statistics rather than for the F statistics themselves.
asymptotic	logical. If set to TRUE the asymptotic (chi-square) distribution instead of the exact (F) distribution will be used to compute the p values (only if pval is TRUE).
	currently not used.

### Value

an object of class "ts" with the same time properties as the time series in x

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### See Also

```
Fstats, plot. Fstats
```

### **Examples**

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data("nhtemp")
## plot the data
plot(nhtemp)

## test the model null hypothesis that the average temperature remains
## constant over the years for potential break points between 1941
## (corresponds to from = 0.5) and 1962 (corresponds to to = 0.85)
## compute F statistics
fs <- Fstats(nhtemp ~ 1, from = 0.5, to = 0.85)
## plot the p values without boundary
plot(fs, pval = TRUE, alpha = 0.01)
## add the boundary in another colour
lines(boundary(fs, pval = TRUE, alpha = 0.01), col = 2)</pre>
```

boundary.mefp

Boundary Function for Monitoring of Structural Changes

### **Description**

Computes boundary for an object of class "mefp"

### Usage

```
## S3 method for class 'mefp'
boundary(x, ...)
```

### **Arguments**

```
x an object of class "mefp".... currently not used.
```

# Value

an object of class "ts" with the same time properties as the monitored process

#### See Also

```
mefp, plot.mefp
```

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### **Examples**

breakdates

Breakdates Corresponding to Breakpoints

### **Description**

A generic function for computing the breakdates corresponding to breakpoints (and their confidence intervals).

### Usage

```
breakdates(obj, format.times = FALSE, ...)
```

### **Arguments**

obj	An object of class "breakpoints", "breakpointsfull" or their confidence intervals as returned by confint.
format.times	logical. If set to TRUE a vector of strings with the formatted breakdates. See details for more information.
	currently not used.

### **Details**

Breakpoints are the number of observations that are the last in one segment and breakdates are the corresponding points on the underlying time scale. The breakdates can be formatted which enhances readability in particular for quarterly or monthly time series. For example the breakdate 2002.75 of a monthly time series will be formatted to "2002(10)".

# Value

A vector or matrix containing the breakdates.

#### See Also

```
breakpoints, confint
```

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### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)
bp.nile <- breakpoints(Nile ~ 1)</pre>
summary(bp.nile)
plot(bp.nile)
## compute breakdates corresponding to the
## breakpoints of minimum BIC segmentation
breakdates(bp.nile)
## confidence intervals
ci.nile <- confint(bp.nile)</pre>
breakdates(ci.nile)
ci.nile
plot(Nile)
lines(ci.nile)
```

breakfactor

Factor Coding of Segmentations

### **Description**

Generates a factor encoding the segmentation given by a set of breakpoints.

### Usage

```
breakfactor(obj, breaks = NULL, labels = NULL, ...)
```

# Arguments

ob	j	An object of class "breakpoints" or "breakpointsfull" respectively.
br	eaks	an integer specifying the number of breaks to extract (only if obj is of class "breakpointsfull"), by default the minimum BIC partition is used.
la	bels	a vector of labels for the returned factor, by default the segments are numbered starting from "segment1".
		further arguments passed to factor.

### Value

A factor encoding the segmentation.

### See Also

breakpoints

### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)

## compute breakpoints
bp.nile <- breakpoints(Nile ~ 1)

## fit and visualize segmented and unsegmented model
fm0 <- lm(Nile ~ 1)
fm1 <- lm(Nile ~ breakfactor(bp.nile, breaks = 1))

lines(fitted(fm0), col = 3)
lines(fitted(fm1), col = 4)
lines(bp.nile, breaks = 1)</pre>
```

breakpoints

Dating Breaks

### **Description**

Computation of breakpoints in regression relationships. Given a number of breaks the function computes the optimal breakpoints.

#### Usage

```
## S3 method for class 'formula'
breakpoints(formula, h = 0.15, breaks = c("BIC", "LWZ", "RSS", "all"),
    data = list(), hpc = c("none", "foreach"), ...)
## S3 method for class 'matrix'
breakpoints(obj, y, h = 0.15, breaks = c("BIC", "LWZ", "RSS", "all"),
    hpc = c("none", "foreach"), ...)
## S3 method for class 'breakpointsfull'
breakpoints(obj, breaks = c("BIC", "LWZ", "RSS", "all"), ...)
## S3 method for class 'breakpointsfull'
summary(object, breaks = NULL, sort = NULL,
    format.times = NULL, ...)
## S3 method for class 'breakpoints'
lines(x, breaks = NULL, lty = 2, ...)
## S3 method for class 'breakpointsfull'
coef(object, breaks = NULL, names = NULL, ...)
## S3 method for class 'breakpointsfull'
fitted(object, breaks = NULL, bp = NULL, ...)
## S3 method for class 'breakpointsfull'
residuals(object, breaks = NULL, ...)
## S3 method for class 'breakpointsfull'
```

```
vcov(object, breaks = NULL, names = NULL,
het.reg = TRUE, het.err = TRUE, vcov. = NULL, sandwich = TRUE, ...)
```

#### **Arguments**

obj, object an object of class "breakpointsfull", or (only in breakpoints.matrix) re-

gression matrix X.

y response vector.

formula a symbolic description for the model in which breakpoints will be estimated.

h minimal segment size either given as fraction relative to the sample size or as an

integer giving the minimal number of observations in each segment.

breaks either a positive integer specifying the maximal number of breaks to be calcu-

lated, or a string specifying the information criterion to use to automatically determine the optimal number of breaks (see also logLik). "all" means the maximal number allowed by h is used. NULL is treated as the default of the

breakpoints function (i.e. BIC).

data an optional data frame containing the variables in the model. By default the

variables are taken from the environment which breakpoints is called from.

hpc a character specifying the high performance computing support. Default is

"none", can be set to "foreach".

... arguments passed to recresid.

sort logical. If set to TRUE summary tries to match the breakpoints from partitions

with different numbers of breaks. The default tries to sort if a suitable matching

can be found.

format.times logical. If set to TRUE a vector of strings with the formatted breakdates is printed.

See breakdates for more information.

x an object of class "breakpoints".

1ty line type.

names a character vector giving the names of the segments. If of length 1 it is taken to

be a generic prefix, e.g. "segment".

bp integer vector denoting the breakpoint indices for which to get the fitted values.

Default is to choose according to breaks. If both are given, bp takes precedence.

het.reg logical. Should heterogeneous regressors be assumed? If set to FALSE the dis-

tribution of the regressors is assumed to be homogeneous over the segments.

het.err logical. Should heterogeneous errors be assumed? If set to FALSE the distribu-

tion of the errors is assumed to be homogeneous over the segments.

vcov. a function to extract the covariance matrix for the coefficients of a fitted model

of class "lm".

sandwich logical. Is the function vcov. the sandwich estimator or only the middle part?

#### **Details**

All procedures in this package are concerned with testing or assessing deviations from stability in the classical linear regression model

$$y_i = x_i^{\top} \beta + u_i$$

In many applications it is reasonable to assume that there are m breakpoints, where the coefficients shift from one stable regression relationship to a different one. Thus, there are m+1 segments in which the regression coefficients are constant, and the model can be rewritten as

$$y_i = x_i^{\top} \beta_j + u_i$$
  $(i = i_{j-1} + 1, \dots, i_j, j = 1, \dots, m+1)$ 

where j denotes the segment index. In practice the breakpoints  $i_j$  are rarely given exogenously, but have to be estimated. breakpoints estimates these breakpoints by minimizing the residual sum of squares (RSS) of the equation above.

The foundation for estimating breaks in time series regression models was given by Bai (1994) and was extended to multiple breaks by Bai (1997ab) and Bai & Perron (1998). breakpoints implements the algorithm described in Bai & Perron (2003) for simultaneous estimation of multiple breakpoints. The distribution function used for the confidence intervals for the breakpoints is given in Bai (1997b). The ideas behind this implementation are described in Zeileis et al. (2003).

The algorithm for computing the optimal breakpoints given the number of breaks is based on a dynamic programming approach. The underlying idea is that of the Bellman principle. The main computational effort is to compute a triangular RSS matrix, which gives the residual sum of squares for a segment starting at observation i and ending at i' with i < i'.

Given a formula as the first argument, breakpoints computes an object of class "breakpointsfull" which inherits from "breakpoints". This contains in particular the triangular RSS matrix and functions to extract an optimal segmentation. A summary of this object will give the breakpoints (and associated) breakdates for all segmentations up to the maximal number of breaks together with the associated RSS, BIC and LWZ. These will be plotted if plot is applied and thus visualize the minimum BIC and LWZ estimators of the number of breakpoints. From an object of class "breakpointsfull" an arbitrary number of breaks (admissible by the minimum segment size h) can be extracted by another application of breakpoints, returning an object of class "breakpoints". This contains only the breakpoints for the specified number of breaks and some model properties (number of observations, regressors, time series properties and the associated RSS) but not the triangular RSS matrix and related extractor functions. The set of breakpoints which is associated by default with a "breakpointsfull" object is the minimum BIC partition.

Breakpoints are the number of observations that are the last in one segment, it is also possible to compute the corresponding breakdates which are the breakpoints on the underlying time scale. The breakdates can be formatted which enhances readability in particular for quarterly or monthly time series. For example the breakdate 2002.75 of a monthly time series will be formatted to "2002(10)". See breakdates for more details.

From a "breakpointsfull" object confidence intervals for the breakpoints can be computed using the method of confint. The breakdates corresponding to the breakpoints can again be computed by breakdates. The breakpoints and their confidence intervals can be visualized by lines. Convenience functions are provided for extracting the coefficients and covariance matrix, fitted values and residuals of segmented models.

The log likelihood as well as some information criteria can be computed using the methods for the logLik, AIC and LWZ. As for linear models the log likelihood is computed on a normal model and the degrees of freedom are the number of regression coefficients multiplied by the number of segments plus the number of estimated breakpoints plus 1 for the error variance. More details can be found on the help page of the method logLik.breakpoints.

As the maximum of a sequence of F statistics is equivalent to the minimum OLS estimator of the breakpoint in a 2-segment partition it can be extracted by breakpoints from an object of class "Fstats" as computed by Fstats. However, this cannot be used to extract a larger number of breakpoints.

For illustration see the commented examples below and Zeileis et al. (2003).

Optional support for high performance computing is available, currently using foreach for the dynamic programming algorithm. If hpc = "foreach" is to be used, a parallel backend should be registered before. See foreach for more information.

#### Value

An object of class "breakpoints" is a list with the following elements:

**breakpoints** the breakpoints of the optimal partition with the number of breaks specified (set to NA if the optimal 1-segment solution is reported),

**RSS** the associated RSS,

**nobs** the number of observations,

nreg the number of regressors,

call the function call,

datatsp the time series properties tsp of the data, if any, c(1/nobs, 1, nobs) otherwise.

If applied to a formula as first argument, breakpoints returns an object of class "breakpointsfull" (which inherits from "breakpoints"), that contains some additional (or slightly different) elements such as:

**breakpoints** the breakpoints of the minimum BIC partition,

**RSS** a function which takes two arguments i, j and computes the residual sum of squares for a segment starting at observation i and ending at j by looking up the corresponding element in the triangular RSS matrix RSS.triang,

**RSS.triang** a list encoding the triangular RSS matrix.

#### References

Bai J. (1994), Least Squares Estimation of a Shift in Linear Processes, *Journal of Time Series Analysis*, **15**, 453-472.

Bai J. (1997a), Estimating Multiple Breaks One at a Time, Econometric Theory, 13, 315-352.

Bai J. (1997b), Estimation of a Change Point in Multiple Regression Models, *Review of Economics and Statistics*, **79**, 551-563.

Bai J., Perron P. (1998), Estimating and Testing Linear Models With Multiple Structural Changes, *Econometrica*, **66**, 47-78.

Bai J., Perron P. (2003), Computation and Analysis of Multiple Structural Change Models, *Journal of Applied Econometrics*, **18**, 1-22.

Zeileis A., Kleiber C., Krämer W., Hornik K. (2003), Testing and Dating of Structural Changes in Practice, *Computational Statistics and Data Analysis*, **44**, 109-123. doi:10.1016/S0167-9473(03)00030-6.

Zeileis A., Shah A., Patnaik I. (2010), Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes, *Computational Statistics and Data Analysis*, **54**(6), 1696–1706. doi:10.1016/j.csda.2009.12.005.

### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)
## F statistics indicate one breakpoint
fs.nile <- Fstats(Nile ~ 1)</pre>
plot(fs.nile)
breakpoints(fs.nile)
lines(breakpoints(fs.nile))
bp.nile <- breakpoints(Nile ~ 1)</pre>
summary(bp.nile)
## the BIC and LWZ also choose one breakpoint
plot(bp.nile)
breakpoints(bp.nile)
breakpoints(bp.nile, breaks = "LWZ")
## fit null hypothesis model and model with 1 breakpoint
fm0 <- lm(Nile ~ 1)
fm1 <- lm(Nile ~ breakfactor(bp.nile, breaks = 1))</pre>
lines(ts(fitted(fm0), start = 1871), col = 3)
lines(ts(fitted(fm1), start = 1871), col = 4)
lines(bp.nile)
## confidence interval
ci.nile <- confint(bp.nile)</pre>
ci.nile
lines(ci.nile)
## UK Seatbelt data: a SARIMA(1,0,0)(1,0,0)_12 model
## (fitted by OLS) is used and reveals (at least) two
## breakpoints - one in 1973 associated with the oil crisis and
## one in 1983 due to the introduction of compulsory
## wearing of seatbelts in the UK.
data("UKDriverDeaths")
seatbelt <- log10(UKDriverDeaths)</pre>
seatbelt <- cbind(seatbelt, lag(seatbelt, k = -1), lag(seatbelt, k = -12))</pre>
```

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```
colnames(seatbelt) \leftarrow c("y", "ylag1", "ylag12")
seatbelt <- window(seatbelt, start = c(1970, 1), end = c(1984, 12))
plot(seatbelt[,"y"], ylab = expression(log[10](casualties)))
## testing
re.seat <- efp(y ~ ylag1 + ylag12, data = seatbelt, type = "RE")
plot(re.seat)
## dating
bp.seat <- breakpoints(y ~ ylag1 + ylag12, data = seatbelt, h = 0.1)</pre>
summary(bp.seat)
lines(bp.seat, breaks = 2)
## minimum BIC partition
plot(bp.seat)
breakpoints(bp.seat)
## the BIC would choose 0 breakpoints although the RE and supF test
## clearly reject the hypothesis of structural stability. Bai &
## Perron (2003) report that the BIC has problems in dynamic regressions.
## due to the shape of the RE process of the F statistics choose two
## breakpoints and fit corresponding models
bp.seat2 <- breakpoints(bp.seat, breaks = 2)</pre>
fm0 <- lm(y \sim ylag1 + ylag12, data = seatbelt)
fm1 \leftarrow lm(y \sim breakfactor(bp.seat2)/(ylag1 + ylag12) - 1, data = seatbelt)
## plot
plot(seatbelt[,"y"], ylab = expression(log[10](casualties)))
time.seat <- as.vector(time(seatbelt))</pre>
lines(time.seat, fitted(fm0), col = 3)
lines(time.seat, fitted(fm1), col = 4)
lines(bp.seat2)
## confidence intervals
ci.seat2 <- confint(bp.seat, breaks = 2)</pre>
ci.seat2
lines(ci.seat2)
```

catL2BB

Generators for efpFunctionals along Categorical Variables

### **Description**

Generators for efpFunctional objects suitable for aggregating empirical fluctuation processes to test statistics along (ordinal) categorical variables.

# Usage

```
catL2BB(freq)
ordL2BB(freq, nproc = NULL, nrep = 1e5, probs = c(0:84/100, 850:1000/1000), ...)
ordwmax(freq, algorithm = mvtnorm::GenzBretz(), ...)
```

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#### **Arguments**

freq	object specifying the category frequencies for the categorical variable to be used for aggregation: either a gefp object, a factor, or a numeric vector with either absolute or relative category frequencies.
nproc	numeric. Number of processes used for simulating from the asymptotic distribution (passed to efpFunctional). If feq is a gefp object, then its number of processes is used by default.
nrep	numeric. Number of replications used for simulating from the asymptotic distribution (passed to efpFunctional).
probs	numeric vector specifying for which probabilities critical values should be tabulated.
	further arguments passed to efpFunctional.
algorithm	algorithm specification passed to pmvnorm for computing the asymptotic distribution.

#### **Details**

Merkle, Fan, and Zeileis (2014) discuss three functionals that are suitable for aggregating empirical fluctuation processes along categorical variables, especially ordinal variables. The functions catL2BB, ordL2BB, and ordwmax all require a specification of the relative frequencies within each category (which can be computed from various specifications, see arguments). All of them employ efpFunctional (Zeileis 2006) internally to set up an object that can be employed with gefp fluctuation processes.

catL2BB results in a chi-squared test. This is essentially the LM test counterpart to the likelihood ratio test that assesses a split into unordered categories.

ordL2BB is the ordinal counterpart to supLM where aggregation is done along the ordered categories (rather than continuously). The asymptotic distribution is non-standard and needs to be simulated for every combination of frequencies and number of processes. Hence, this is somewhat more time-consuming compared to the closed-form solution employed in catL2BB. It is also possible to store the result of ordL2BB in case it needs to be applied several gefp fluctuation processes.

ordwmax is a weighted double maximum test based on ideas previously suggested by Hothorn and Zeileis (2008) in the context of maximally selected statistics. The asymptotic distribution is (multivariate) normal and computed by means of pmvnorm.

#### Value

An object of class efpFunctional.

#### References

Hothorn T., Zeileis A. (2008), Generalized Maximally Selected Statistics. *Biometrics*, **64**, 1263–1269.

Merkle E.C., Fan J., Zeileis A. (2014), Testing for Measurement Invariance with Respect to an Ordinal Variable. *Psychometrika*, **79**(4), 569–584. doi:10.1007/S11336-013-9376-7.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

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### See Also

```
efpFunctional, gefp
```

### **Examples**

```
## artificial data
set.seed(1)
d <- data.frame(</pre>
  x = runif(200, -1, 1),
  z = factor(rep(1:4, each = 50)),
  err = rnorm(200)
d$y <- rep(c(0.5, -0.5), c(150, 50)) * d$x + d$err
## empirical fluctuation process
scus <- gefp(y \sim x, data = d, fit = lm, order.by = \sim z)
## chi-squared-type test (unordered LM-type test)
LMuo <- catL2BB(scus)
plot(scus, functional = LMuo)
sctest(scus, functional = LMuo)
## ordinal maxLM test (with few replications only to save time)
maxLMo <- ordL2BB(scus, nrep = 10000)</pre>
plot(scus, functional = maxLMo)
sctest(scus, functional = maxLMo)
## ordinal weighted double maximum test
WDM <- ordwmax(scus)</pre>
plot(scus, functional = WDM)
sctest(scus, functional = WDM)
```

confint.breakpointsfull

Confidence Intervals for Breakpoints

### **Description**

Computes confidence intervals for breakpoints.

### Usage

```
## S3 method for class 'breakpointsfull'
confint(object, parm = NULL, level = 0.95,
    breaks = NULL, het.reg = TRUE, het.err = TRUE, vcov. = NULL, sandwich = TRUE, ...)
## S3 method for class 'confint.breakpoints'
lines(x, col = 2, angle = 90, length = 0.05,
    code = 3, at = NULL, breakpoints = TRUE, ...)
```

#### **Arguments**

object an object of class "breakpointsfull" as computed by breakpoints from a formula. the same as breaks, only one of the two should be specified. parm level the confidence level required. breaks an integer specifying the number of breaks to be used. By default the breaks of the minimum BIC partition are used. het.reg logical. Should heterogeneous regressors be assumed? If set to FALSE the distribution of the regressors is assumed to be homogeneous over the segments. logical. Should heterogeneous errors be assumed? If set to FALSE the distribuhet.err tion of the errors is assumed to be homogeneous over the segments. a function to extract the covariance matrix for the coefficients of a fitted model vcov. of class "lm". sandwich logical. Is the function vcov. the sandwich estimator or only the middle part? an object of class "confint.breakpoints" as returned by confint. col, angle, length, code arguments passed to arrows. position on the y axis, where the confidence arrows should be drawn. By default at they are drawn at the bottom of the plot. logical. If TRUE vertical lines for the breakpoints are drawn. breakpoints

#### **Details**

As the breakpoints are integers (observation numbers) the corresponding confidence intervals are also rounded to integers.

currently not used.

The distribution function used for the computation of confidence intervals of breakpoints is given in Bai (1997). The procedure, in particular the usage of heterogeneous regressors and/or errors, is described in more detail in Bai & Perron (2003).

The breakpoints should be computed from a formula with breakpoints, then the confidence intervals for the breakpoints can be derived by confint and these can be visualized by lines. For an example see below.

#### Value

A matrix containing the breakpoints and their lower and upper confidence boundary for the given level.

#### References

Bai J. (1997), Estimation of a Change Point in Multiple Regression Models, *Review of Economics and Statistics*, **79**, 551-563.

Bai J., Perron P. (2003), Computation and Analysis of Multiple Structural Change Models, *Journal of Applied Econometrics*, **18**, 1-22.

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### See Also

breakpoints

### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)

## dating breaks
bp.nile <- breakpoints(Nile ~ 1)
ci.nile <- confint(bp.nile, breaks = 1)
lines(ci.nile)</pre>
```

DJIA

Dow Jones Industrial Average

### Description

Weekly closing values of the Dow Jones Industrial Average.

### Usage

```
data("DJIA")
```

### **Format**

A weekly univariate time series of class "zoo" from 1971-07-01 to 1974-08-02.

#### **Source**

```
Appendix A in Hsu (1979).
```

### References

Hsu D. A. (1979), Detecting Shifts of Parameter in Gamma Sequences with Applications to Stock Price and Air Traffic Flow Analysis, *Journal of the American Statistical Association*, **74**, 31–40.

# **Examples**

```
data("DJIA")
## look at log-difference returns
djia <- diff(log(DJIA))
plot(djia)
## convenience functions
## set up a normal regression model which
## explicitely also models the variance</pre>
```

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```
normlm <- function(formula, data = list()) {</pre>
  rval <- lm(formula, data = data)</pre>
  class(rval) <- c("normlm", "lm")</pre>
  return(rval)
}
estfun.normlm <- function(obj) {</pre>
  res <- residuals(obj)</pre>
  ef <- NextMethod(obj)</pre>
  sigma2 <- mean(res^2)</pre>
  rval <- cbind(ef, res^2 - sigma2)</pre>
  colnames(rval) <- c(colnames(ef), "(Variance)")</pre>
  return(rval)
}
## normal model (with constant mean and variance) for log returns
m1 <- gefp(djia ~ 1, fit = normlm, vcov = meatHAC, sandwich = FALSE)</pre>
plot(m1, aggregate = FALSE)
## suggests a clear break in the variance (but not the mean)
bp <- breakpoints(I(djia^2) ~ 1)</pre>
plot(bp)
## -> clearly one break
time(djia)[bp$breakpoints]
## visualization
plot(djia)
abline(v = time(djia)[bp$breakpoints], lty = 2)
lines(time(djia)[confint(bp)$confint[c(1,3)]], rep(min(djia), 2), col = 2, type = "b", pch = 3)
```

durab

US Labor Productivity

### **Description**

US labor productivity in the manufacturing/durables sector.

### Usage

```
data("durab")
```

### **Format**

durab is a multivariate monthly time series from 1947(3) to 2001(4) with variables

**y** growth rate of the Industrial Production Index to average weekly labor hours in the manufacturing/durables sector,

```
lag lag 1 of the series y,
```

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#### **Source**

The data set is available from Bruce Hansen's homepage https://www.ssc.wisc.edu/~bhansen/. For more information see Hansen (2001).

#### References

Hansen B. (2001), The New Econometrics of Structural Change: Dating Breaks in U.S. Labor Productivity, *Journal of Economic Perspectives*, **15**, 117–128.

Zeileis A., Leisch F., Kleiber C., Hornik K. (2005), Monitoring Structural Change in Dynamic Econometric Models, *Journal of Applied Econometrics*, **20**, 99–121.

### **Examples**

```
data("durab")
## use AR(1) model as in Hansen (2001) and Zeileis et al. (2005)
durab.model <- y ~ lag
## historical tests
## OLS-based CUSUM process
ols <- efp(durab.model, data = durab, type = "OLS-CUSUM")
plot(ols)
## F statistics
fs <- Fstats(durab.model, data = durab, from = 0.1)</pre>
plot(fs)
## F statistics based on heteroskadisticy-consistent covariance matrix
fsHC <- Fstats(durab.model, data = durab, from = 0.1,
       vcov = function(x, ...) vcovHC(x, type = "HC", ...))
plot(fsHC)
## monitoring
Durab <- window(durab, start=1964, end = c(1979, 12))
ols.efp <- efp(durab.model, type = "OLS-CUSUM", data = Durab)
newborder <- function(k) 1.723 * k/192
ols.mefp <- mefp(ols.efp, period=2)</pre>
ols.mefp2 <- mefp(ols.efp, border=newborder)</pre>
Durab <- window(durab, start=1964)</pre>
ols.mon <- monitor(ols.mefp)</pre>
ols.mon2 <- monitor(ols.mefp2)</pre>
plot(ols.mon)
lines(boundary(ols.mon2), col = 2)
## Note: critical value for linear boundary taken from Table III
## in Zeileis et al. 2005: (1.568 + 1.896)/2 = 1.732 is a linear
## interpolation between the values for T = 2 and T = 3 at
## alpha = 0.05. A typo switched 1.732 to 1.723.
## dating
bp <- breakpoints(durab.model, data = durab)</pre>
```

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```
summary(bp)
plot(summary(bp))

plot(ols)
lines(breakpoints(bp, breaks = 1), col = 3)
lines(breakpoints(bp, breaks = 2), col = 4)
plot(fs)
lines(breakpoints(bp, breaks = 1), col = 3)
lines(breakpoints(bp, breaks = 2), col = 4)
```

efp

**Empirical Fluctuation Processes** 

### **Description**

Computes an empirical fluctuation process according to a specified method from the generalized fluctuation test framework, which includes CUSUM and MOSUM tests based on recursive or OLS residuals, parameter estimates or ML scores (OLS first order conditions).

### Usage

### **Arguments**

X, y, formula	specification of the linear regression model: either by a regressor matrix X and a response variable y, or by a formula.
data	an optional data frame containing the variables in the model. By default the variables are taken from the environment which efp is called from.
type	specifies which type of fluctuation process will be computed, the default is "Rec-CUSUM". For details see below.
h	bandwidth (for MOSUM and ME processes only), either specified relative to the sample size as a numeric from interval $(0,1)$ , or as an integer $>= 1$ determining the absolute bandwidth size (number of samples).
dynamic	logical. If TRUE the lagged observations are included as a regressor.
rescale	logical. If TRUE the estimates will be standardized by the regressor matrix of the corresponding subsample according to Kuan & Chen (1994); if FALSE the whole regressor matrix will be used. (only if type is either "RE" or "ME")

1rvar logical or character. Should a long-run variance estimator be used for the resid-

uals? By default, the standard OLS variance is employed. Alternatively, <code>lrvar</code> can be used. If <code>lrvar</code> is character ("Andrews" or "Newey-West"), then the corresponding type of long-run variance is used. (The argument is ignored for the

score-based tests where gefp should be used instead.)

vcov a function to extract the covariance matrix for the coefficients of the fitted model

(only for "RE" and "ME").

... currently not used.

#### **Details**

If type is one of "Rec-CUSUM", "OLS-CUSUM", "Rec-MOSUM" or "OLS-MOSUM" the function efp will return a one-dimensional empirical process of sums of residuals. Either it will be based on recursive residuals or on OLS residuals and the process will contain CUmulative SUMs or MOving SUMs of residuals in a certain data window. For the MOSUM and ME processes all estimations are done for the observations in a moving data window, whose size is determined by h and which is shifted over the whole sample.

If type is either "RE" or "ME" a *k*-dimensional process will be returned, if *k* is the number of regressors in the model, as it is based on recursive OLS estimates of the regression coefficients or moving OLS estimates respectively. The recursive estimates test is also called fluctuation test, therefore setting type to "fluctuation" was used to specify it in earlier versions of strucchange. It still can be used now, but will be forced to "RE".

If type is "Score-CUSUM" or "Score-MOSUM" a k+1-dimensional process will be returned, one for each score of the regression coefficients and one for the scores of the variance. The process gives the decorrelated cumulative sums of the ML scores (in a Gaussian model) or first order conditions respectively (in an OLS framework).

If there is a single structural change point  $t^*$ , the recursive CUSUM path starts to depart from its mean 0 at  $t^*$ . The Brownian bridge type paths will have their respective peaks around  $t^*$ . The Brownian bridge increments type paths should have a strong change at  $t^*$ .

The function plot has a method to plot the empirical fluctuation process; with sctest the corresponding test on structural change can be performed.

#### Value

efp returns a list of class "efp" with components including:

process the fitted empirical fluctuation process of class "ts" or "mts" respectively,

type a string with the type of the process fitted,

nreg the number of regressors,
nobs the number of observations,

par the bandwidth h used.

#### References

Brown R.L., Durbin J., Evans J.M. (1975), Techniques for testing constancy of regression relationships over time, *Journal of the Royal Statistical Society*, B, **37**, 149-163.

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Chu C.-S., Hornik K., Kuan C.-M. (1995), MOSUM tests for parameter constancy, *Biometrika*, **82**, 603-617.

Chu C.-S., Hornik K., Kuan C.-M. (1995), The moving-estimates test for parameter stability, *Econometric Theory*, **11**, 669-720.

Hansen B. (1992), Testing for Parameter Instability in Linear Models, *Journal of Policy Modeling*, **14**, 517-533.

Hjort N.L., Koning A. (2002), Tests for Constancy of Model Parameters Over Time, *Nonparametric Statistics*, **14**, 113-132.

Krämer W., Ploberger W., Alt R. (1988), Testing for structural change in dynamic models, *Econometrica*, **56**, 1355-1369.

Kuan C.-M., Hornik K. (1995), The generalized fluctuation test: A unifying view, *Econometric Reviews*, **14**, 135 - 161.

Kuan C.-M., Chen (1994), Implementing the fluctuation and moving estimates tests in dynamic econometric models, *Economics Letters*, **44**, 235-239.

Ploberger W., Krämer W. (1992), The CUSUM test with OLS residuals, *Econometrica*, **60**, 271-285.

Zeileis A., Leisch F., Hornik K., Kleiber C. (2002), strucchange: An R Package for Testing for Structural Change in Linear Regression Models, *Journal of Statistical Software*, **7**(2), 1-38. doi:10.18637/jss.v007.i02.

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

Zeileis A., Hornik K. (2007), Generalized M-Fluctuation Tests for Parameter Instability, *Statistica Neerlandica*, **61**, 488–508. doi:10.1111/j.14679574.2007.00371.x.

#### See Also

```
gefp, plot.efp, print.efp, sctest.efp, boundary.efp
```

### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)

## test the null hypothesis that the annual flow remains constant
## over the years
## compute OLS-based CUSUM process and plot
## with standard and alternative boundaries
ocus.nile <- efp(Nile ~ 1, type = "OLS-CUSUM")
plot(ocus.nile)
plot(ocus.nile, alpha = 0.01, alt.boundary = TRUE)
## calculate corresponding test statistic
sctest(ocus.nile)</pre>
```

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```
## UK Seatbelt data: a SARIMA(1,0,0)(1,0,0)_12 model
## (fitted by OLS) is used and reveals (at least) two
## breakpoints - one in 1973 associated with the oil crisis and
## one in 1983 due to the introduction of compulsory
## wearing of seatbelts in the UK.
data("UKDriverDeaths")
seatbelt <- log10(UKDriverDeaths)</pre>
seatbelt <- cbind(seatbelt, lag(seatbelt, k = -1), lag(seatbelt, k = -12))
colnames(seatbelt) <- c("y", "ylag1", "ylag12")</pre>
seatbelt <- window(seatbelt, start = c(1970, 1), end = c(1984, 12))
plot(seatbelt[,"y"], ylab = expression(log[10](casualties)))
## use RE process
re.seat <- efp(y ~ ylag1 + ylag12, data = seatbelt, type = "RE")
plot(re.seat)
plot(re.seat, functional = NULL)
sctest(re.seat)
```

efpFunctional

Functionals for Fluctuation Processes

### **Description**

Computes an object for aggregating, plotting and testing empirical fluctuation processes.

#### Usage

```
efpFunctional(functional = list(comp = function(x) max(abs(x)), time = max),
boundary = function(x) rep(1, length(x)),
computePval = NULL, computeCritval = NULL,
plotProcess = NULL, lim.process = "Brownian bridge",
nobs = 10000, nrep = 50000, nproc = 1:20, h = 0.5,
probs = c(0:84/100, 850:1000/1000))
```

### **Arguments**

functional either a function for aggregating fluctuation processes or a list with two func-

tions names "comp" and "time".

boundary a boundary function.

computePval a function for computing p values. If neither computePval nor computeCritval

are specified critical values are simulated with settings as specified below.

computeCritval a function for computing critical values. If neither computePval nor computeCritval

are specified critical values are simulated with settings as specified below.

plotProcess a function for plotting the empirical process, if set to NULL a suitable function is

set up.

lim.process a string specifying the limiting process.

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nobs integer specifying the number of observations of each Brownian motion simulated.

nrep integer specifying the number of replications.

nproc integer specifying for which number of processes Brownian motions should be simulated. If set to NULL only nproc = 1 is used and all other values are derived from a Bonferroni correction.

h bandwidth parameter for increment processes.

probs numeric vector specifying for which probabilities critical values should be tab-

ulated.

#### **Details**

efpFunctional computes an object of class "efpFunctional" which then knows how to do inference based on empirical fluctuation processes (currently only for gefp objects and not yet for efp objects) and how to visualize the corresponding processes.

efpFunctionals for many frequently used test statistics are provided: maxBB for the double maximum statistic, meanL2BB for the Cramer-von Mises statistic, or rangeBB for the range statistic. Furthermore, supLM generates an object of class "efpFunctional" for a certain trimming parameter, see the examples. More details can be found in Zeileis (2006). Based on Merkle, Fan, and Zeileis (2014), further efpFunctional generators for aggregating along (ordered) categorical variables have been added: catL2BB, ordL2BB, ordwmax.

For setting up an efpFunctional, the functions computeStatistic, computePval, and plotProcess need to be supplied. These should have the following interfaces: computeStatistic should take a single argument which is the process itself, i.e., essentially a n x k matrix where n is the number of observations and k the number of processes (regressors). computePval should take two arguments: a scalar test statistic and the number of processes k. plotProcess should take two arguments: an object of class "gefp" and alpha the level of significance for any boundaries or critical values to be visualized.

#### Value

efpFunctional returns a list of class "efpFunctional" with components including:

plotProcess a function for plotting empirical fluctuation processes, computeStatistic

a function for computing a test statistic from an empirical fluctuation process,

computePval a function for computing the corresponding p value,

computeCritval a function for computing critical values.

#### References

Merkle E.C., Zeileis A. (2013), Tests of Measurement Invariance without Subgroups: A Generalization of Classical Methods. *Psychometrika*, **78**(1), 59–82. doi:10.1007/S11336-012-9302-4

Merkle E.C., Fan J., Zeileis A. (2014), Testing for Measurement Invariance with Respect to an Ordinal Variable. *Psychometrika*, **79**(4), 569–584. doi:10.1007/S11336-013-9376-7.

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

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Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

Zeileis A., Hornik K. (2007), Generalized M-Fluctuation Tests for Parameter Instability, *Statistica Neerlandica*, **61**, 488–508. doi:10.1111/j.1467-9574.2007.00371.x.

#### See Also

```
gefp, supLM, catL2BB, sctest.default
```

### **Examples**

```
data("BostonHomicide")
gcus <- gefp(homicides ~ 1, family = poisson, vcov = kernHAC,</pre>
  data = BostonHomicide)
plot(gcus, functional = meanL2BB)
sctest(gcus, functional = meanL2BB)
y <- rnorm(1000)
x1 <- runif(1000)
x2 <- runif(1000)
## supWald statistic computed by Fstats()
fs <- Fstats(y \sim x1 + x2, from = 0.1)
plot(fs)
sctest(fs)
## compare with supLM statistic
scus <- gefp(y \sim x1 + x2, fit = lm)
plot(scus, functional = supLM(0.1))
sctest(scus, functional = supLM(0.1))
## seatbelt data
data("UKDriverDeaths")
seatbelt <- log10(UKDriverDeaths)</pre>
seatbelt <- cbind(seatbelt, lag(seatbelt, k = -1), lag(seatbelt, k = -12))</pre>
colnames(seatbelt) <- c("y", "ylag1", "ylag12")</pre>
seatbelt <- window(seatbelt, start = c(1970, 1), end = c(1984, 12))
scus.seat <- gefp(y ~ ylag1 + ylag12, data = seatbelt)</pre>
## double maximum test
plot(scus.seat)
## range test
plot(scus.seat, functional = rangeBB)
## Cramer-von Mises statistic (Nyblom-Hansen test)
plot(scus.seat, functional = meanL2BB)
## supLM test
plot(scus.seat, functional = supLM(0.1))
```

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Fstats	F Statistics	

### Description

Computes a series of F statistics for a specified data window.

# Usage

```
Fstats(formula, from = 0.15, to = NULL, data = list(), vcov. = NULL)
```

### **Arguments**

<b>6</b>	
formula	a symbolic description for the model to be tested
from, to	numeric. If from is smaller than 1 they are interpreted as percentages of data and by default to is taken to be 1 - from. F statistics will be calculated for the observations (n*from):(n*to), when n is the number of observations in the model. If from is greater than 1 it is interpreted to be the index and to defaults to n - from. If from is a vector with two elements, then from and to are interpreted as time specifications like in ts, see also the examples.
data	an optional data frame containing the variables in the model. By default the variables are taken from the environment which Fstats is called from.
vcov.	a function to extract the covariance matrix for the coefficients of a fitted model of class "1m".

#### **Details**

For every potential change point in from: to a F statistic (Chow test statistic) is computed. For this an OLS model is fitted for the observations before and after the potential change point, i.e. 2k parameters have to be estimated, and the error sum of squares is computed (ESS). Another OLS model for all observations with a restricted sum of squares (RSS) is computed, hence k parameters have to be estimated here. If n is the number of observations and k the number of regressors in the model, the formula is:

$$F = \frac{(RSS - ESS)}{ESS/(n - 2k)}$$

Note that this statistic has an asymptotic chi-squared distribution with k degrees of freedom and (under the assumption of normality) F/k has an exact F distribution with k and n - 2k degrees of freedom.

### Value

Fstats returns an object of class "Fstats", which contains mainly a time series of F statistics. The function plot has a method to plot the F statistics or the corresponding p values; with sctest a supF-, aveF- or expF-test on structural change can be performed.

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#### References

Andrews D.W.K. (1993), Tests for parameter instability and structural change with unknown change point, *Econometrica*, **61**, 821-856.

Hansen B. (1992), Tests for parameter instability in regressions with I(1) processes, *Journal of Business & Economic Statistics*, **10**, 321-335.

Hansen B. (1997), Approximate asymptotic p values for structural-change tests, *Journal of Business & Economic Statistics*, **15**, 60-67.

#### See Also

```
plot.Fstats, sctest.Fstats, boundary.Fstats
```

### **Examples**

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)
## test the null hypothesis that the annual flow remains constant
## over the years
fs.nile <- Fstats(Nile ~ 1)</pre>
plot(fs.nile)
sctest(fs.nile)
## visualize the breakpoint implied by the argmax of the F statistics
plot(Nile)
lines(breakpoints(fs.nile))
## UK Seatbelt data: a SARIMA(1,0,0)(1,0,0)_12 model
## (fitted by OLS) is used and reveals (at least) two
## breakpoints - one in 1973 associated with the oil crisis and
## one in 1983 due to the introduction of compulsory
## wearing of seatbelts in the UK.
data("UKDriverDeaths")
seatbelt <- log10(UKDriverDeaths)</pre>
seatbelt <- cbind(seatbelt, lag(seatbelt, k = -1), lag(seatbelt, k = -12))
colnames(seatbelt) <- c("y", "ylag1", "ylag12")</pre>
seatbelt \leftarrow window(seatbelt, start = c(1970, 1), end = c(1984,12))
plot(seatbelt[,"y"], ylab = expression(log[10](casualties)))
## compute F statistics for potential breakpoints between
## 1971(6) (corresponds to from = 0.1) and 1983(6) (corresponds to
## to = 0.9 = 1 - from, the default)
## compute F statistics
fs <- Fstats(y ~ ylag1 + ylag12, data = seatbelt, from = 0.1)
## this gives the same result
fs <- Fstats(y \sim ylag1 + ylag12, data = seatbelt, from = c(1971, 6),
             to = c(1983, 6))
## plot the F statistics
plot(fs, alpha = 0.01)
## plot F statistics with aveF boundary
```

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```
plot(fs, aveF = TRUE)
## perform the expF test
sctest(fs, type = "expF")
```

gefp

Generalized Empirical M-Fluctuation Processes

# Description

Computes an empirical M-fluctuation process from the scores of a fitted model.

### Usage

```
gefp(..., fit = glm, scores = estfun, vcov = NULL,
  decorrelate = TRUE, sandwich = TRUE, order.by = NULL,
  fitArgs = NULL, parm = NULL, data = list())
```

# Arguments

•••	specification of some model which is passed together with data to the fit function: $fm < -fit(, data = data)$ . If fit is set to NULL the first argument is assumed to be already the fitted model $fm$ (all other arguments in are ignored and a warning is issued in this case).
fit	a model fitting function, typically lm, glm or rlm.
scores	a function which extracts the scores or estimating function from the fitted object: $scores(fm)$ .
vcov	a function to extract the covariance matrix for the coefficients of the fitted model: $vcov(fm, order.by = order.by, data = data)$ .
decorrelate	logical. Should the process be decorrelated?
sandwich	logical. Is the function vcov the full sandwich estimator or only the meat?
order.by	Either a vector z or a formula with a single explanatory variable like $\sim$ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).
fitArgs	List of additional arguments which could be passed to the fit function. Usually, this is not needed and will be sufficient to pass arguments to fit.
parm	integer or character specifying the component of the estimating functions which should be used (by default all components are used).
data	an optional data frame containing the variables in the specification and the order.by model. By default the variables are taken from the environment which gefp is called from.
	fit scores vcov decorrelate sandwich order.by fitArgs parm

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#### Value

gefp returns a list of class "gefp" with components including:

process the fitted empirical fluctuation process of class "zoo",

nreg the number of regressors, nobs the number of observations,

fit the fit function used,

scores the scores function used,

fitted.model the fitted model.

### References

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

Zeileis A., Hornik K. (2007), Generalized M-Fluctuation Tests for Parameter Instability, *Statistica Neerlandica*, **61**, 488–508. doi:10.1111/j.1467-9574.2007.00371.x.

Zeileis A., Shah A., Patnaik I. (2010), Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes, *Computational Statistics and Data Analysis*, **54**(6), 1696–1706. doi:10.1016/j.csda.2009.12.005.

#### See Also

```
efp, efpFunctional
```

### **Examples**

GermanM1

German M1 Money Demand

### **Description**

German M1 money demand.

### Usage

```
data("GermanM1")
```

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#### **Format**

GermanM1 is a data frame containing 12 quarterly time series from 1961(1) to 1995(4) and two further variables. historyM1 is the subset of GermanM1 up to 1990(2), i.e., the data before the German monetary unification on 1990-06-01. monitorM1 is the complement of historyM1, i.e., the data after the unification. All three data frames contain the variables

```
m time series. Logarithm of real M1 per capita,
```

- p time series. Logarithm of a price index,
- y time series. Logarithm of real per capita gross national product,
- R time series. Long-run interest rate,
- dm time series. First differences of m,
- dy2 time series. First differences of lag 2 of y,
- dR time series. First differences of R,
- dR1 time series. First differences of lag 1 of R,
- dp time series. First differences of p,
- m1 time series. Lag 1 of m,
- y1 time series. Lag 1 of y,
- R1 time series. Lag 1 of R,

season factor coding the seasonality,

**ecm.res** vector containing the OLS residuals of the Lütkepohl et al. (1999) model fitted in the history period.

### **Details**

Lütkepohl et al. (1999) investigate the linearity and stability of German M1 money demand: they find a stable regression relation for the time before the monetary union on 1990-06-01 but a clear structural instability afterwards.

Zeileis et al. (2005) use a model with ecm. res instead of m1, y1 and R1, which leads to equivalent results in the history period but slightly different results in the monitoring period. The reason for the replacement is that stationary regressors are needed for the structural change tests. See references and the examples below for more details.

#### Source

The data is provided by the German central bank and is available online in the data archive of the Journal of Applied Econometrics http://qed.econ.queensu.ca/jae/1999-v14.5/lutkepohl-terasvirta-wolters/.

#### References

Lütkepohl H., Teräsvirta T., Wolters J. (1999), Investigating Stability and Linearity of a German M1 Money Demand Function, *Journal of Applied Econometrics*, **14**, 511-525.

Zeileis A., Leisch F., Kleiber C., Hornik K. (2005), Monitoring Structural Change in Dynamic Econometric Models, *Journal of Applied Econometrics*, **20**, 99–121.

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### **Examples**

```
data("GermanM1")
## Lütkepohl et al. (1999) use the following model
LTW.model <- dm \sim dy2 + dR + dR1 + dp + m1 + y1 + R1 + season
## Zeileis et al. (2005) use
M1.model \leftarrow dm \sim dy2 + dR + dR1 + dp + ecm.res + season
## historical tests
ols <- efp(LTW.model, data = GermanM1, type = "OLS-CUSUM")
plot(ols)
re <- efp(LTW.model, data = GermanM1, type = "fluctuation")</pre>
fs <- Fstats(LTW.model, data = GermanM1, from = 0.1)</pre>
plot(fs)
## monitoring
M1 <- historyM1
ols.efp <- efp(M1.model, type = "OLS-CUSUM", data = M1)
newborder <- function(k) 1.5778*k/118
ols.mefp <- mefp(ols.efp, period = 2)</pre>
ols.mefp2 <- mefp(ols.efp, border = newborder)</pre>
M1 <- GermanM1
ols.mon <- monitor(ols.mefp)</pre>
ols.mon2 <- monitor(ols.mefp2)</pre>
plot(ols.mon)
lines(boundary(ols.mon2), col = 2)
## dating
bp <- breakpoints(LTW.model, data = GermanM1)</pre>
summary(bp)
plot(bp)
plot(fs)
lines(confint(bp))
```

Grossarl

Marriages, Births and Deaths in Grossarl

# Description

Data about the number of marriages, illegitimate and legitimate births, and deaths in the Austrian Alpine village Grossarl during the 18th and 19th century.

### Usage

```
data("Grossarl")
```

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#### **Format**

Grossar1 is a data frame containing 6 annual time series (1700 - 1899), 3 factors coding policy interventions and 1 vector with the year (plain numeric).

marriages time series. Number of marriages,

illegitimate time series. Number of illegitimate births,

legitimate time series. Number of legitimate births,

legitimate time series. Number of deaths,

fraction time series. Fraction of illegitimate births,

lag.marriages time series. Number of marriages in the previous year,

politics ordered factor coding 4 different political regimes,

morals ordered factor coding 5 different moral regulations,

nuptiality ordered factor coding 5 different marriage restrictions,

year numeric. Year of observation.

#### **Details**

The data frame contains historical demographic data from Grossarl, a village in the Alpine region of Salzburg, Austria, during the 18th and 19th century. During this period, the total population of Grossarl did not vary much on the whole, with the very exception of the period of the protestant emigrations in 1731/32.

Especially during the archbishopric, moral interventions aimed at lowering the proportion of illegitimate baptisms. For details see the references.

#### Source

Parish registers provide the basic demographic series of baptisms and burials (which is almost equivalent to births and deaths in the study area) and marriages. For more information see Veichtlbauer et al. (2006).

### References

Veichtlbauer O., Zeileis A., Leisch F. (2006), The Impact Of Policy Interventions on a Pre-Industrial Population System in the Austrian Alps, forthcoming.

Zeileis A., Veichtlbauer O. (2002), Policy Interventions Affecting Illegitimacy in Preindustrial Austria: A Structural Change Analysis, In R. Dutter (ed.), *Festschrift 50 Jahre Österreichische Statistische Gesellschaft*, 133-146, Österreichische Statistische Gesellschaft.

### **Examples**

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```
plot.type = "single", col = grey(c(0.7, 0, 0)), lty = c(1, 1, 3),
  lwd = 1.5, ylab = "annual Grossarl series"))
legend("topright", c("deaths", "births", "marriages"), col = grey(c(0.7, 0, 0)),
  lty = c(1, 1, 3), bty = "n")
## illegitimate births
##########################
## lm + MOSUM
plot(Grossarl$fraction)
fm.min <- lm(fraction ~ politics, data = Grossarl)</pre>
fm.ext <- lm(fraction ~ politics + morals + nuptiality + marriages,</pre>
  data = Grossarl)
lines(ts(fitted(fm.min), start = 1700), col = 2)
lines(ts(fitted(fm.ext), start = 1700), col = 4)
mos.min <- efp(fraction ~ politics, data = Grossarl, type = "OLS-MOSUM")</pre>
mos.ext <- efp(fraction ~ politics + morals + nuptiality + marriages,</pre>
  data = Grossarl, type = "OLS-MOSUM")
plot(mos.min)
lines(mos.ext, lty = 2)
## dating
bp <- breakpoints(fraction ~ 1, data = Grossarl, h = 0.1)</pre>
summary(bp)
## RSS, BIC, AIC
plot(bp)
plot(0:8, AIC(bp), type = "b")
## probably use 5 or 6 breakpoints and compare with
## coding of the factors as used by us
##
## politics
                              1803
                                        1816 1850
## morals
              1736 1753 1771 1803
## nuptiality
                              1803 1810 1816
                                                   1883
##
## m = 5
                   1753 1785
                                        1821 1856 1878
## m = 6
             1734 1754 1785
                                       1821 1856 1878
               6 2 5
                                        1 4 3
## fitted models
coef(bp, breaks = 6)
plot(Grossarl$fraction)
lines(fitted(bp, breaks = 6), col = 2)
lines(ts(fitted(fm.ext), start = 1700), col = 4)
## marriages
############
## 1m + MOSUM
plot(Grossarl$marriages)
fm.min <- lm(marriages ~ politics, data = Grossarl)</pre>
fm.ext <- lm(marriages ~ politics + morals + nuptiality, data = Grossarl)</pre>
lines(ts(fitted(fm.min), start = 1700), col = 2)
lines(ts(fitted(fm.ext), start = 1700), col = 4)
```

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```
mos.min <- efp(marriages ~ politics, data = Grossarl, type = "OLS-MOSUM")</pre>
mos.ext <- efp(marriages ~ politics + morals + nuptiality, data = Grossarl,</pre>
  type = "OLS-MOSUM")
plot(mos.min)
lines(mos.ext, lty = 2)
bp <- breakpoints(marriages ~ 1, data = Grossarl, h = 0.1)</pre>
summary(bp)
## RSS, BIC, AIC
plot(bp)
plot(0:8, AIC(bp), type = "b")
## probably use 3 or 4 breakpoints and compare with
## coding of the factors as used by us
##
## politics
                               1803
                                         1816 1850
## morals
            1736 1753 1771 1803
## nuptiality
                              1803 1810 1816
                                                   1883
## m = 3
               1738
                                         1813
                                                   1875
                         1794
## m = 4
               1738
                                         1814
                                                   1875
##
                2
                          4
                                          1
                                                    3
## fitted models
coef(bp, breaks = 4)
plot(Grossarl$marriages)
lines(fitted(bp, breaks = 4), col = 2)
lines(ts(fitted(fm.ext), start = 1700), col = 4)
```

logLik.breakpoints Log Likelihood and Information Criteria for Breakpoints

# Description

Computation of log likelihood and AIC type information criteria for partitions given by breakpoints.

### Usage

```
## S3 method for class 'breakpointsfull'
logLik(object, breaks = NULL, ...)
## S3 method for class 'breakpointsfull'
AIC(object, breaks = NULL, ..., k = 2)
## S3 method for class 'breakpointsfull'
LWZ(object, ...)
## S3 method for class 'breakpoints'
LWZ(object, ...)
```

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# **Arguments**

object	an object of class "breakpoints" or "breakpointsfull".
breaks	if object is of class "breakpointsfull" the number of breaks can be specified.
•••	for LWZ, parameters passed to AIC.breakpointsfull. Unused in logLik and AIC.
k	the penalty parameter to be used, the default $k = 2$ is the classical AIC, $k = log(n)$ gives the BIC, $k = 0.299 * log(n)^2.1$ gives the LWZ, if n is the number of observations.

#### **Details**

As for linear models the log likelihood is computed on a normal model and the degrees of freedom are the number of regression coefficients multiplied by the number of segments plus the number of estimated breakpoints plus 1 for the error variance.

If AIC or LWZ is applied to an object of class "breakpointsfull" breaks can be a vector of integers and the AIC or LWZ for each corresponding partition will be returned. By default the maximal number of breaks stored in the object is used. See below for an example.

#### Value

An object of class "logLik" or a simple vector containing the AIC respectively.

#### References

Liu, J., Wu, S., & Zidek, J. V. (1997). On segmented multivariate regression. *Statistica Sinica*, 497-525.

#### See Also

breakpoints

```
## Nile data with one breakpoint: the annual flows drop in 1898
## because the first Ashwan dam was built
data("Nile")
plot(Nile)

bp.nile <- breakpoints(Nile ~ 1)
summary(bp.nile)
plot(bp.nile)

## BIC of partitions with 0 to 5 breakpoints
plot(0:5, AIC(bp.nile, k = log(bp.nile$nobs)), type = "b")
## AIC
plot(0:5, AIC(bp.nile), type = "b")
## LWZ
plot(0:5, LWZ(bp.nile), type = "b")</pre>
```

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```
## BIC, AIC, LWZ, log likelihood of a single partition
bp.nile1 <- breakpoints(bp.nile, breaks = 1)
AIC(bp.nile1, k = log(bp.nile1$nobs))
AIC(bp.nile1)
LWZ(bp.nile1)
logLik(bp.nile1)</pre>
```

magnitude

Magnitudes of Breakpoints

# Description

Computes magnitude statistics for breakpoints.

# Usage

```
## $3 method for class 'breakpointsfull'
magnitude(object,
  interval = 0.1, breaks = NULL, component = "trend", ...)
```

# **Arguments**

object an object of class "breakpointsfull" as computed by breakpoints. interval the (one-side) interval over which to compute the magnitude for the

the (one-side) interval over which to compute the magnitude for the columns RMSD and MAD. If below 1, fraction of the time series length, otherwise number

of observations.

breaks how many breaks to use or which statistic to use for estimating it, see breakpoints

for details. If NULL, uses the defaults.

component which covariate(s) used for fitting the breakpointsfull object to use for mag-

nitude estimation. The components are additive.

... ignored

#### **Details**

The breakpoint magnitude is estimated using several statistics: the difference (diff) in the fitted value immediately before and after the break, and the root mean squared deviation (RMSD), mean absolute deviation (MAD) and mean deviation (MD) between the fitted values of the model preceding the break extrapolated over interval samples after the break, and vice versa. RMSD and MAD should be more robust estimators of magnitude compared to diff.

# Value

A list with the following elements, compatible with the magnitude format as output by bfast:

**Mag** a matrix containing the magnitude estimates (in columns) for each breakpoint (in rows), **m.x** sample number at which the largest break was detected, twice

y.x fitted values before and after the largest break

Magnitude the maximum diff magnitude of the largest break

**Time** time of the largest break (same as m. x, but single value)

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# See Also

breakpoints

#### **Examples**

```
data(Nile)

trend <- 1:length(Nile)
bp.nile <- breakpoints(Nile ~ trend)

# The trend component is default, set "component" to the
# name of your coviariate(s), if it is different.
magnitude(bp.nile)</pre>
```

mefp

Monitoring of Empirical Fluctuation Processes

# **Description**

Online monitoring of structural breaks in a linear regression model. A sequential fluctuation test based on parameter estimates or OLS residuals signals structural breaks.

# Usage

```
## S3 method for class 'formula'
mefp(formula, type = c("OLS-CUSUM", "OLS-MOSUM", "RE", "ME",
    "fluctuation"), data, h = 1, alpha = 0.05,
    functional = c("max", "range"), period = 10,
    tolerance = .Machine$double.eps^0.5, CritvalTable = NULL,
    rescale = NULL, border = NULL, ...)
## S3 method for class 'matrix'
mefp(obj, y, type = c("OLS-CUSUM", "OLS-MOSUM", "RE", "ME",
    "fluctuation"), h = 1, alpha = 0.05,
    functional = c("max", "range"), period = 10,
    tolerance = .Machine$double.eps^0.5, CritvalTable = NULL,
    rescale = NULL, border = NULL, ...)
## S3 method for class 'efp'
mefp(obj, alpha=0.05, functional = c("max", "range"),
    period = 10, tolerance = .Machine$double.eps^0.5,
    CritvalTable = NULL, rescale = NULL, border = NULL, ...)
monitor(obj, data = NULL, verbose = TRUE)
monitor.matrix(obj, X, y, verbose = TRUE)
```

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#### **Arguments**

formula specification of the linear regression model by a formula.

obj object of class "efp" (for mefp), "mefp" (for monitor), or the regression matrix

X (for mefp.matrix.

X regression matrix X.
y response vector.

data an optional data frame containing the variables in the model. By default the

variables are taken from the environment which efp is called from.

type specifies which type of fluctuation process will be computed.

h (only used for MOSUM/ME processes). A numeric scalar from interval (0,1)

specifying the size of the data window relative to the sample size.

alpha Significance level of the test, i.e., probability of type I error.

functional Determines if maximum or range of parameter differences is used as statistic.

period (only used for MOSUM/ME processes). Maximum time (relative to the history

period) that will be monitored. Default is 10 times the history period.

tolerance Tolerance for numeric == comparisons.

CritvalTable Table of critical values, this table is interpolated to get critical values for ar-

bitrary alphas. The default depends on the type of fluctuation process (precomputed tables are available for all types). *This argument is under develop-*

ment.

rescale If TRUE the estimates will be standardized by the regressor matrix of the cor-

responding subsample similar to Kuan & Chen (1994); if FALSE the historic regressor matrix will be used. The default is to rescale the monitoring processes

of type "ME" but not of "RE".

border An optional user-specified border function for the empirical process. *This argu-*

ment is under development.

verbose If TRUE, signal breaks by text output.

... Currently not used.

## Details

mefp creates an object of class "mefp" either from a model formula or from an object of class "efp". In addition to the arguments of efp, the type of statistic and a significance level for the monitoring must be specified. The monitoring itself is performed by monitor, which can be called arbitrarily often on objects of class "mefp". If new data have arrived, then the empirical fluctuation process is computed for the new data. If the process crosses the boundaries corresponding to the significance level alpha, a structural break is detected (and signaled).

The typical usage is to initialize the monitoring by creation of an object of class "mefp" either using a formula or an "efp" object. Data available at this stage are considered the *history sample*, which is kept fixed during the complete monitoring process, and may not contain any structural changes.

Subsequent calls to monitor perform a sequential test of the null hypothesis of no structural change in new data against the general alternative of changes in one or more of the coefficients of the regression model.

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The recursive estimates test is also called fluctuation test, therefore setting type to "fluctuation" was used to specify it in earlier versions of strucchange. It still can be used now, but will be forced to "RE"

#### Value

An object of class mefp that includes components such as:

breakpoint index of the detected breakpoint, NA otherwise,

process a matrix denoting the coefficients of the empirical fluctuation process,

computeEmpProc a function to compute the empirical process based on newcoef, Q and k for

ME/RE and X and y for OLS-MOSUM/OLS-CUSUM.

last number of observations,

type a string with the type of the process fitted,

alpha the significance level.

#### References

Leisch F., Hornik K., Kuan C.-M. (2000), Monitoring Structural Changes with the Generalized Fluctuation Test, *Econometric Theory*, **16**, 835–854.

Zeileis A., Leisch F., Kleiber C., Hornik K. (2005), Monitoring Structural Change in Dynamic Econometric Models, *Journal of Applied Econometrics*, **20**, 99–121. doi:10.1002/jae.776.

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

Zeileis A., Shah A., Patnaik I. (2010), Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes, *Computational Statistics and Data Analysis*, **54**(6), 1696–1706. doi:10.1016/j.csda.2009.12.005.

#### See Also

```
plot.mefp, boundary.mefp
```

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```
# and now monitor on all data
me3 <- monitor(me2, data=df1)</pre>
plot(me3)
## Load dataset "USIncExp" with income and expenditure in the US
## and choose a suitable subset for the history period
data("USIncExp")
USIncExp3 <- window(USIncExp, start=c(1969,1), end=c(1971,12))</pre>
## initialize the monitoring with the formula interface
me.mefp <- mefp(expenditure~income, type="ME", rescale=TRUE,</pre>
                    data=USIncExp3, alpha=0.05)
## monitor the new observations for the year 1972
USIncExp3 <- window(USIncExp, start=c(1969,1), end=c(1972,12))</pre>
me.mefp <- monitor(me.mefp)</pre>
## monitor the new data for the years 1973-1976
USIncExp3 <- window(USIncExp, start=c(1969,1), end=c(1976,12))</pre>
me.mefp <- monitor(me.mefp)</pre>
plot(me.mefp, functional = NULL)
```

PhillipsCurve

UK Phillips Curve Equation Data

# Description

Macroeconomic time series from the United Kingdom with variables for estimating the Phillips curve equation.

# Usage

```
data("PhillipsCurve")
```

## **Format**

A multivariate annual time series from 1857 to 1987 with the columns

- **p** Logarithm of the consumer price index,
- w Logarithm of nominal wages,
- u Unemployment rate,
- **dp** First differences of p,
- dw First differences of w,
- du First differences of u
- u1 Lag 1 of u,
- dp1 Lag 1 of dp.

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# **Source**

The data is available online in the data archive of the Journal of Applied Econometrics <a href="http://ged.econ.queensu.ca/jae/2003-v18.1/bai-perron/">http://ged.econ.queensu.ca/jae/2003-v18.1/bai-perron/</a>.

# References

Alogoskoufis G.S., Smith R. (1991), The Phillips Curve, the Persistence of Inflation, and the Lucas Critique: Evidence from Exchange Rate Regimes, *American Economic Review*, **81**, 1254-1275.

Bai J., Perron P. (2003), Computation and Analysis of Multiple Structural Change Models, *Journal of Applied Econometrics*, **18**, 1-22.

```
## load and plot data
data("PhillipsCurve")
uk <- window(PhillipsCurve, start = 1948)</pre>
plot(uk[, "dp"])
## AR(1) inflation model
## estimate breakpoints
bp.inf <- breakpoints(dp ~ dp1, data = uk, h = 8)</pre>
plot(bp.inf)
summary(bp.inf)
## fit segmented model with three breaks
fac.inf <- breakfactor(bp.inf, breaks = 2, label = "seg")
fm.inf <- lm(dp \sim 0 + fac.inf/dp1, data = uk)
summary(fm.inf)
## Results from Table 2 in Bai & Perron (2003):
## coefficient estimates
coef(bp.inf, breaks = 2)
## corresponding standard errors
sqrt(sapply(vcov(bp.inf, breaks = 2), diag))
## breakpoints and confidence intervals
confint(bp.inf, breaks = 2)
## Phillips curve equation
## estimate breakpoints
bp.pc <- breakpoints(dw ~ dp1 + du + u1, data = uk, h = 5, breaks = 5)</pre>
## look at RSS and BIC
plot(bp.pc)
summary(bp.pc)
## fit segmented model with three breaks
fac.pc <- breakfactor(bp.pc, breaks = 2, label = "seg")</pre>
fm.pc \leftarrow lm(dw \sim 0 + fac.pc/dp1 + du + u1, data = uk)
summary(fm.pc)
## Results from Table 3 in Bai & Perron (2003):
## coefficient estimates
```

plot.efp

```
coef(fm.pc)
## corresponding standard errors
sqrt(diag(vcov(fm.pc)))
## breakpoints and confidence intervals
confint(bp.pc, breaks = 2, het.err = FALSE)
```

plot.efp

Plot Empirical Fluctuation Process

# **Description**

Plot and lines method for objects of class "efp"

# Usage

```
## S3 method for class 'efp'
plot(x, alpha = 0.05, alt.boundary = FALSE, boundary = TRUE,
    functional = "max", main = NULL, ylim = NULL,
    ylab = "Empirical fluctuation process", ...)
## S3 method for class 'efp'
lines(x, functional = "max", ...)
```

high-level plot function parameters.

# **Arguments**

х	an object of class "efp".
alpha	numeric from interval (0,1) indicating the confidence level for which the boundary of the corresponding test will be computed.
alt.boundary	logical. If set to TRUE alternative boundaries (instead of the standard linear boundaries) will be plotted (for CUSUM processes only).
boundary	logical. If set to FALSE the boundary will be computed but not plotted.
functional	indicates which functional should be applied to the process before plotting and which boundaries should be used. If set to NULL a multiple process with boundaries for the "max" functional is plotted. For more details see below.
main, ylim, ylab	,

# **Details**

Plots are available for the "max" functional for all process types. For Brownian bridge type processes the maximum or mean squared Euclidean norm ("maxL2" and "meanL2") can be used for aggregating before plotting. No plots are available for the "range" functional.

Alternative boundaries that are proportional to the standard deviation of the corresponding limiting process are available for processes with Brownian motion or Brownian bridge limiting processes.

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#### Value

efp returns an object of class "efp" which inherits from the class "ts" or "mts" respectively. The function plot has a method to plot the empirical fluctuation process; with sctest the corresponding test for structural change can be performed.

#### References

Brown R.L., Durbin J., Evans J.M. (1975), Techniques for testing constancy of regression relationships over time, *Journal of the Royal Statistical Society*, B, **37**, 149-163.

Chu C.-S., Hornik K., Kuan C.-M. (1995), MOSUM tests for parameter constancy, *Biometrika*, **82**, 603-617.

Chu C.-S., Hornik K., Kuan C.-M. (1995), The moving-estimates test for parameter stability, *Econometric Theory*, **11**, 669-720.

Krämer W., Ploberger W., Alt R. (1988), Testing for structural change in dynamic models, *Econometrica*, **56**, 1355-1369.

Kuan C.-M., Hornik K. (1995), The generalized fluctuation test: A unifying view, *Econometric Reviews*, **14**, 135 - 161.

Kuan C.-M., Chen (1994), Implementing the fluctuation and moving estimates tests in dynamic econometric models, *Economics Letters*, **44**, 235-239.

Ploberger W., Krämer W. (1992), The CUSUM test with OLS residuals, *Econometrica*, **60**, 271-285.

Zeileis A., Leisch F., Hornik K., Kleiber C. (2002), strucchange: An R Package for Testing for Structural Change in Linear Regression Models, *Journal of Statistical Software*, **7**(2), 1-38. doi:10.18637/jss.v007.i02.

Zeileis A. (2004), Alternative Boundaries for CUSUM Tests, Statistical Papers, 45, 123-131.

#### See Also

```
efp, boundary.efp, sctest.efp
```

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data("nhtemp")
## plot the data
plot(nhtemp)

## test the model null hypothesis that the average temperature remains
## constant over the years
## compute Rec-CUSUM fluctuation process
temp.cus <- efp(nhtemp ~ 1)
## plot the process
plot(temp.cus, alpha = 0.01)
## and calculate the test statistic
sctest(temp.cus)

## compute (recursive estimates) fluctuation process</pre>
```

plot.Fstats

```
## with an additional linear trend regressor
lin.trend <- 1:60
temp.me <- efp(nhtemp ~ lin.trend, type = "fluctuation")
## plot the bivariate process
plot(temp.me, functional = NULL)
## and perform the corresponding test
sctest(temp.me)</pre>
```

 ${\tt plot.Fstats}$ 

Plot F Statistics

# **Description**

Plotting method for objects of class "Fstats"

# Usage

```
## S3 method for class 'Fstats'
plot(x, pval = FALSE, asymptotic = FALSE, alpha = 0.05,
    boundary = TRUE, aveF = FALSE, xlab = "Time", ylab = NULL,
    ylim = NULL, ...)
```

# Arguments

x	an object of class "Fstats".
pval	logical. If set to TRUE the corresponding $\boldsymbol{p}$ values instead of the original $\boldsymbol{F}$ statistics will be plotted.
asymptotic	logical. If set to TRUE the asymptotic (chi-square) distribution instead of the exact (F) distribution will be used to compute the p values (only if pval is TRUE).
alpha	numeric from interval $(0,1)$ indicating the confidence level for which the boundary of the supF test will be computed.
boundary	logical. If set to FALSE the boundary will be computed but not plotted.
aveF	logical. If set to TRUE the boundary of the aveF test will be plotted. As this is a boundary for the mean of the F statistics rather than for the F statistics themselves a dashed line for the mean of the F statistics will also be plotted.
xlab, ylab, ylim	
	high-level plot function parameters.

# Value

No return value, called for side effects.

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# References

Andrews D.W.K. (1993), Tests for parameter instability and structural change with unknown change point, *Econometrica*, **61**, 821-856.

Hansen B. (1992), Tests for parameter instability in regressions with I(1) processes, *Journal of Business & Economic Statistics*, **10**, 321-335.

Hansen B. (1997), Approximate asymptotic p values for structural-change tests, *Journal of Business & Economic Statistics*, **15**, 60-67.

#### See Also

```
Fstats, boundary. Fstats, sctest. Fstats
```

# **Examples**

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data("nhtemp")
## plot the data
plot(nhtemp)

## test the model null hypothesis that the average temperature remains
## constant over the years for potential break points between 1941
## (corresponds to from = 0.5) and 1962 (corresponds to to = 0.85)
## compute F statistics
fs <- Fstats(nhtemp ~ 1, from = 0.5, to = 0.85)
## plot the F statistics
plot(fs, alpha = 0.01)
## and the corresponding p values
plot(fs, pval = TRUE, alpha = 0.01)
## perform the aveF test
sctest(fs, type = "aveF")</pre>
```

plot.mefp

Plot Methods for mefp Objects

# **Description**

This is a method of the generic plot function for "mefp" objects as returned by mefp or monitor. It plots the empirical fluctuation process (or a functional thereof) as a time series plot, and includes boundaries corresponding to the significance level of the monitoring procedure.

# Usage

```
## S3 method for class 'mefp'
plot(x, boundary = TRUE, functional = "max", main = NULL,
    ylab = "Empirical fluctuation process", ylim = NULL, ...)
```

48 RealInt

# **Arguments**

x an object of class "mefp".

boundary if FALSE, plotting of boundaries is suppressed.

functional indicates which functional should be applied to a multivariate empirical process.

If set to NULL all dimensions of the process (one process per coefficient in the

linear model) are plotted.

main, ylab, ylim, ...

high-level plot function parameters.

#### Value

No return value, called for side effects.

#### See Also

mefp

# **Examples**

RealInt

US Ex-post Real Interest Rate

# Description

US ex-post real interest rate: the three-month treasury bill deflated by the CPI inflation rate.

# Usage

```
data("RealInt")
```

# Format

A quarterly time series from 1961(1) to 1986(3).

# Source

The data is available online in the data archive of the Journal of Applied Econometrics http://qed.econ.queensu.ca/jae/2003-v18.1/bai-perron/.

recresid 49

#### References

Bai J., Perron P. (2003), Computation and Analysis of Multiple Structural Change Models, *Journal of Applied Econometrics*, **18**, 1-22.

Zeileis A., Kleiber C. (2005), Validating Multiple Structural Change Models - A Case Study. Journal of Applied Econometrics, **20**, 685-690.

# **Examples**

```
## load and plot data
data("RealInt")
plot(RealInt)
## estimate breakpoints
bp.ri <- breakpoints(RealInt ~ 1, h = 15)</pre>
plot(bp.ri)
summary(bp.ri)
## fit segmented model with three breaks
fac.ri <- breakfactor(bp.ri, breaks = 3, label = "seg")</pre>
fm.ri <- lm(RealInt ~ 0 + fac.ri)</pre>
summary(fm.ri)
## setup kernel HAC estimator
vcov.ri <- function(x, ...) kernHAC(x, kernel = "Quadratic Spectral",</pre>
  prewhite = 1, approx = ^{"AR(1)"}, ...)
## Results from Table 1 in Bai & Perron (2003):
## coefficient estimates
coef(bp.ri, breaks = 3)
## corresponding standard errors
sapply(vcov(bp.ri, breaks = 3, vcov = vcov.ri), sqrt)
## breakpoints and confidence intervals
confint(bp.ri, breaks = 3, vcov = vcov.ri)
## Visualization
plot(RealInt)
lines(as.vector(time(RealInt)), fitted(fm.ri), col = 4)
lines(confint(bp.ri, breaks = 3, vcov = vcov.ri))
```

recresid

Recursive Residuals

# Description

A generic function for computing the recursive residuals (standardized one step prediction errors) of a linear regression model.

50 recresid

# Usage

```
## Default S3 method:
recresid(x, y, start = ncol(x) + 1, end = nrow(x),
   tol = sqrt(.Machine$double.eps)/ncol(x), qr.tol = 1e-7, ...)
## S3 method for class 'formula'
recresid(formula, data = list(), ...)
## S3 method for class 'lm'
recresid(x, data = list(), ...)
```

# **Arguments**

x, y, formula	specification of the linear regression model: either by a regressor matrix x and a response variable y, or by a formula or by a fitted object x of class "lm".
start, end	integer. Index of the first and last observation, respectively, for which recursive residuals should be computed. By default, the maximal range is selected.
tol	numeric. A relative tolerance for precision of recursive coefficient estimates, see details.
qr.tol	numeric. The tolerance passed to lm.fit for detecting linear dependencies.
data	an optional data frame containing the variables in the model. By default the variables are taken from the environment which recresid is called from. Specifying data might also be necessary when applying recresid to a fitted model of class "lm" if this does not contain the regressor matrix and the response.
	currently not used.

## **Details**

Recursive residuals are standardized one-step-ahead prediction errors. Under the usual assumptions for the linear regression model they are (asymptotically) normal and i.i.d. (see Brown, Durbin, Evans, 1975, for details).

The default method computes the initial coefficient estimates via QR decomposition, using lm.fit. In subsequent steps, the updating formula provided by Brown, Durbin, Evans (1975) is employed. To avoid numerical instabilities in the first steps (with typically small sample sizes), the QR solution is computed for comparison. When the relative difference (assessed bey all.equal) between the two solutions falls below tol, only the updating formula is used in subsequent steps.

#### Value

A vector containing the recursive residuals.

# References

Brown R.L., Durbin J., Evans J.M. (1975), Techniques for testing constancy of regression relationships over time, *Journal of the Royal Statistical Society*, B, **37**, 149-163.

# See Also

efp

root.matrix 51

# **Examples**

```
x \leftarrow rnorm(100) + rep(c(0, 2), each = 50)

rr \leftarrow recresid(x \sim 1)

plot(cumsum(rr), type = "l")

plot(efp(x \sim 1, type = "Rec-CUSUM"))
```

root.matrix

Root of a Matrix

# Description

Computes the root of a symmetric and positive semidefinite matrix.

# Usage

```
root.matrix(X)
```

# Arguments

Χ

a symmetric and positive semidefinite matrix

# Value

a symmetric matrix of same dimensions as X

# **Examples**

```
X <- matrix(c(1,2,2,8), ncol=2)
test <- root.matrix(X)
## control results
X
test %*% test</pre>
```

root.matrix.crossprod Root of X^TX

# Description

Computes the root of the Gramian X^TX.

# Usage

```
root.matrix.crossprod(X)
```

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# Arguments

X a matrix

#### Value

a symmetric matrix V where V^2 equals X^TX

# **Examples**

```
set.seed(1)
n <- 100
p <- 3
X <- matrix(rnorm(n*p),nrow=n, ncol=p)
test <- root.matrix.crossprod(X)
## control results
t(X) %*% X
test %*% test</pre>
```

scPublications

Structural Change Publications

# Description

Bibliographic information about papers related to structural change and changepoints published in 27 different econometrics and statistics journals.

# Usage

```
data("scPublications")
```

#### **Format**

A data frame containing information on 835 structural change papers in 9 variables.

```
author character. Author(s) of the paper.
```

title character. Title of the paper.

journal factor. In which journal was the paper published?

**year** numeric. Year of publication.

volume numeric. Journal volume.

issue character. Issue within the journal volume.

bpage numeric. Page on which the paper begins.

epage numeric. Page on which the paper ends.

type factor. Is the journal an econometrics or statistics journal?

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#### **Details**

The data set scPublications includes bibliographic information about publications related to structural change and obtained from the 'ISI Web of Science'. The query was based on the 'Science Citation Index Expanded' and 'Social Sciences Citation Index' (for the full range of years available: 1900-2006 and 1956-2006, respectively). The 'Source Title' was restricted to the 27 journals in the data frame and the 'Topic' to be one of the following: structural change, structural break, structural stability, structural instability, parameter instability, parameter stability, parameter constancy, change point, change-point, breakpoint, break-point, break point, CUSUM, MOSUM. Additionally, the famous CUSUM paper of Brown, Durbin and Evans (1975) was added manually to scPublications (because it did not match the query above).

#### Source

ISI Web of Science at https://www.webofknowledge.com/. Queried by James Bullard.

#### **Examples**

```
## construct time series:
## number of sc publications in econometrics/statistics
data("scPublications")
## select years from 1987 and
## `most important' journals
pub <- scPublications</pre>
pub <- subset(pub, year > 1986)
tab1 <- table(pub$journal)</pre>
nam1 <- names(tab1)[as.vector(tab1) > 9] ## at least 10 papers
tab2 <- sapply(levels(pub$journal), function(x) min(subset(pub, journal == x)$year))
nam2 <- names(tab2)[as.vector(tab2) < 1991] ## started at least in 1990</pre>
nam <- nam1[nam1 %in% nam2]</pre>
pub <- subset(pub, as.character(journal) %in% nam)</pre>
pub$journal <- factor(pub$journal)</pre>
pub_data <- pub</pre>
## generate time series
pub <- with(pub, tapply(type, year, table))</pre>
pub <- zoo(t(sapply(pub, cbind)), 1987:2006)
colnames(pub) <- levels(pub_data$type)</pre>
## visualize
plot(pub, ylim = c(0, 35))
```

sctest

Structural Change Tests

# **Description**

Generic function for performing structural change tests.

54 sctest

#### Usage

```
sctest(x, ...)
```

# **Arguments**

x an object.

... arguments passed to methods.

# **Details**

sctest is a generic function for performing/extracting structural change tests based on various types of objects. The strucchange package provides various types of methods.

First, structural change tests based on F statistics in linear regression models (Fstats), empirical fluctuation processes in linear regression models (efp), and generalized empirical fluctuation processes in parametric models (gefp) are available in the corresponding sctest methods.

Second, convenience interfaces for carrying out structural change tests in linear regression models and general parametric models are provided in sctest.formula and sctest.default, respectively.

#### Value

An object of class "htest" containing:

statistic the test statistic,

p.value the corresponding p value,

method a character string with the method used, data.name a character string with the data name.

# References

Zeileis A., Leisch F., Hornik K., Kleiber C. (2002), strucchange: An R Package for Testing for Structural Change in Linear Regression Models, *Journal of Statistical Software*, **7**(2), 1-38. doi:10.18637/jss.v007.i02.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

# See Also

```
sctest.formula, sctest.default, sctest.Fstats, sctest.efp, sctest.gefp
```

sctest.default 55

sctest.default Structural Change Tests in Parametric Models
-------------------------------------------------------------

# Description

Performs model-based tests for structural change (or parameter instability) in parametric models.

# Usage

```
## Default S3 method:
sctest(x, order.by = NULL, functional = maxBB,
   vcov = NULL, scores = estfun, decorrelate = TRUE, sandwich = TRUE,
   parm = NULL, plot = FALSE, from = 0.1, to = NULL, nobs = NULL,
   nrep = 50000, width = 0.15, xlab = NULL, ...)
```

# Arguments

 Suments	
х	a model object. The model class can in principle be arbitrary but needs to provide suitable methods for extracting the scores and associated variance-covariance matrix vcov.
order.by	either a vector z or a formula with a single explanatory variable like $\sim$ z. The observations in the model are ordered by the size of z. If set to NULL (the default) the observations are assumed to be ordered (e.g., a time series).
functional	either a character specification of the functional to be used or an efpFunctional object. For a list of functionals see the details.
vcov	a function to extract the covariance matrix for the coefficients of the fitted model: vcov(x, order.by = order.by, data = data). Alternatively, the character string "info", for details see below.
scores	a function which extracts the scores or estimating function from the fitted object: scores(x), by default this is estfun.
decorrelate	logical. Should the process be decorrelated?
sandwich	logical. Is the function vcov the full sandwich estimator or only the meat?
parm	integer or character specifying the component of the estimating functions which should be used (by default all components are used).
plot	logical. Should the result of the test also be visualized?
from, to	numeric. In case the functional is "supLM" (or equivalently "maxLM"), from and to can be passed to the supLM functional.
nobs, nrep	numeric. In case the functional is "maxLMo", nobs and nrep are passed to the catL2BB functional.
width	numeric. In case the functional is "MOSUM", the bandwidth width is passed to the $\max$ MOSUM functional.
xlab,	graphical parameters passed to the plot method (in case plot = TRUE).

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#### **Details**

sctest.default is a convenience interface to gefp for structural change tests (or parameter instability tests) in general parametric models. It proceeds in the following steps:

- 1. The generalized empirical fluctuation process (or score-based CUSUM process) is computed via scus <- gefp(x, fit = NULL, ...) where ... comprises the arguments order.by, vcov, scores, decorrelate, sandwich, parm that are simply passed on to gefp.
- 2. The empirical fluctuation process is visualized (if plot = TRUE) via plot(scus, functional = functional, ...).
- 3. The empirical fluctuation is assessed by the corresponding significance test via sctest(scus, functional = functional).

The main motivation for prociding the convenience interface is that these three steps can be easily carried out in one go along with a two convenience options:

- 1. By default, the covariance is computed by an outer-product of gradients estimator just as in gefp. This is always available based on the scores. Additionally, by setting vcov = "info", the corresponding information matrix can be used. Then the average information is assumed to be provided by the vcov method for the model class. (Note that this is only sensible for models estimated by maximum likelihood.)
- 2. Instead of providing the functional by an efpFunctional object, the test labels employed by Merkle and Zeileis (2013) and Merkle, Fan, and Zeileis (2013) can be used for convenience. Namely, for continuous numeric orderings, the following functionals are available: functional = "DM" or "dmax" provides the double-maximum test (maxBB). "CvM" is the Cramer-von Mises functional meanL2BB. "supLM" or equivalently "maxLM" is Andrews' supLM test (supLM). "MOSUM" or "maxMOSUM" is the MOSUM functional (maxMOSUM), and "range" is the range functional rangeBB. Furthermore, several functionals suitable for (ordered) categorical order.by variables are provided: "LMuo" is the unordered LM test (catL2BB), "WDMo" is the weighted double-maximum test for ordered variables (ordwmax), and "maxLMo" is the maxLM test for ordered variables (ordL2BB).

The theoretical model class is introduced in Zeileis and Hornik (2007) with a unifying view in Zeileis (2005), especially from an econometric perspective. Zeileis (2006) introduces the underling computational tools gefp and efpFunctional.

Merkle and Zeileis (2013) discuss the methods in the context of measurement invariance which is particularly relevant to psychometric models for cross section data. Merkle, Fan, and Zeileis (2014) extend the results to ordered categorical variables.

Zeileis, Shah, and Patnaik (2013) provide a unifying discussion in the context of time series methods, specifically in financial econometrics.

#### Value

An object of class "htest" containing:

statistic the test statistic,

p. value the corresponding p value,

method a character string with the method used, data.name a character string with the data name.

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#### References

Merkle E.C., Zeileis A. (2013), Tests of Measurement Invariance without Subgroups: A Generalization of Classical Methods. *Psychometrika*, **78**(1), 59–82. doi:10.1007/S11336-012-9302-4

Merkle E.C., Fan J., Zeileis A. (2014), Testing for Measurement Invariance with Respect to an Ordinal Variable. *Psychometrika*, **79**(4), 569–584. doi:10.1007/S11336-013-9376-7.

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

Zeileis A., Hornik K. (2007), Generalized M-Fluctuation Tests for Parameter Instability, *Statistica Neerlandica*, **61**, 488–508. doi:10.1111/j.1467-9574.2007.00371.x.

Zeileis A., Shah A., Patnaik I. (2010), Testing, Monitoring, and Dating Structural Changes in Exchange Rate Regimes, *Computational Statistics and Data Analysis*, **54**(6), 1696–1706. doi:10.1016/j.csda.2009.12.005.

#### See Also

```
gefp, efpFunctional
```

# **Examples**

```
## Zeileis and Hornik (2007), Section 5.3, Figure 6
data("Grossarl")
m <- glm(cbind(illegitimate, legitimate) ~ 1, family = binomial, data = Grossarl,
    subset = time(fraction) <= 1800)
sctest(m, order.by = 1700:1800, functional = "CvM")</pre>
```

sctest.efp

Generalized Fluctuation Tests

# Description

Performs a generalized fluctuation test.

## Usage

```
## S3 method for class 'efp'
sctest(x, alt.boundary = FALSE,
    functional = c("max", "range", "maxL2", "meanL2"), ...)
```

#### **Arguments**

```
x an object of class "efp".

alt.boundary logical. If set to TRUE alternative boundaries (instead of the standard linear boundaries) will be used (for CUSUM processes only).

functional indicates which functional should be applied to the empirical fluctuation process.

... currently not used.
```

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#### **Details**

The critical values for the MOSUM tests and the ME test are just tabulated for confidence levels between 0.1 and 0.01, thus the p value approximations will be poor for other p values. Similarly the critical values for the maximum and mean squared Euclidean norm ("maxL2" and "meanL2") are tabulated for confidence levels between 0.2 and 0.005.

#### Value

An object of class "htest" containing:

statistic the test statistic,

p.value the corresponding p value,

method a character string with the method used, data.name a character string with the data name.

#### References

Brown R.L., Durbin J., Evans J.M. (1975), Techniques for testing constancy of regression relationships over time, *Journal of the Royal Statistical Society*, B, **37**, 149-163.

Chu C.-S., Hornik K., Kuan C.-M. (1995), MOSUM tests for parameter constancy, *Biometrika*, **82**, 603-617.

Chu C.-S., Hornik K., Kuan C.-M. (1995), The moving-estimates test for parameter stability, *Econometric Theory*, **11**, 669-720.

Krämer W., Ploberger W., Alt R. (1988), Testing for structural change in dynamic models, *Econometrica*, **56**, 1355-1369.

Kuan C.-M., Hornik K. (1995), The generalized fluctuation test: A unifying view, *Econometric Reviews*, **14**, 135 - 161.

Kuan C.-M., Chen (1994), Implementing the fluctuation and moving estimates tests in dynamic econometric models, *Economics Letters*, **44**, 235-239.

Ploberger W., Krämer W. (1992), The CUSUM Test with OLS Residuals, *Econometrica*, **60**, 271-285.

Zeileis A., Leisch F., Hornik K., Kleiber C. (2002), strucchange: An R Package for Testing for Structural Change in Linear Regression Models, *Journal of Statistical Software*, **7**(2), 1-38. doi:10.18637/jss.v007.i02.

Zeileis A. (2004), Alternative Boundaries for CUSUM Tests, Statistical Papers, 45, 123–131.

#### See Also

```
efp, plot.efp
```

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data("nhtemp")
## plot the data
plot(nhtemp)
```

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```
## test the model null hypothesis that the average temperature remains
## constant over the years compute OLS-CUSUM fluctuation process
temp.cus <- efp(nhtemp ~ 1, type = "OLS-CUSUM")
## plot the process with alternative boundaries
plot(temp.cus, alpha = 0.01, alt.boundary = TRUE)
## and calculate the test statistic
sctest(temp.cus)

## compute moving estimates fluctuation process
temp.me <- efp(nhtemp ~ 1, type = "ME", h = 0.2)
## plot the process with functional = "max"
plot(temp.me)
## and perform the corresponding test
sctest(temp.me)</pre>
```

sctest.formula

Structural Change Tests in Linear Regression Models

# **Description**

Performs tests for structural change in linear regression models.

# Usage

```
## S3 method for class 'formula'
sctest(formula, type = , h = 0.15,
    alt.boundary = FALSE, functional = c("max", "range",
    "maxL2", "meanL2"), from = 0.15, to = NULL, point = 0.5,
    asymptotic = FALSE, data, ...)
```

# Arguments

formula	a formula describing the model to be tested.
type	a character string specifying the structural change test that is to be performed, the default is "Rec-CUSUM". Besides the test types described in efp and sctest.Fstats the Chow test and the Nyblom-Hansen test can be performed by setting type to "Chow" or "Nyblom-Hansen", respectively.
h	numeric from interval (0,1) specifying the bandwidth. Determines the size of the data window relative to the sample size (for MOSUM and ME tests only).
alt.boundary	logical. If set to TRUE alternative boundaries (instead of the standard linear boundaries) will be used (for CUSUM processes only).
functional	indicates which functional should be used to aggregate the empirical fluctuation processes to a test statistic.

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from, to	numeric. If from is smaller than 1 they are interpreted as percentages of data and by default to is taken to be the 1 - from. F statistics will be calculated for the observations (n*from): (n*to), when n is the number of observations in the model. If from is greater than 1 it is interpreted to be the index and to defaults to n - from. (for F tests only)
point	parameter of the Chow test for the potential change point. Interpreted analogous to the from parameter. By default taken to be $floor(n*0.5)$ if n is the number of observations in the model.
asymptotic	logical. If TRUE the asymptotic (chi-square) distribution instead of the exact (F) distribution will be used to compute the p value (for Chow test only).
data	an optional data frame containing the variables in the model. By default the variables are taken from the environment which sctest is called from.
	further arguments passed to efp or Fstats.

# **Details**

sctest.formula is a convenience interface for performing structural change tests in linear regression models based on efp and Fstats. It is mainly a wrapper for sctest.efp and sctest.Fstats as it fits an empirical fluctuation process first or computes the F statistics respectively and subsequently performs the corresponding test. The Chow test and the Nyblom-Hansen test are available explicitly here.

An alternative convenience interface for performing structural change tests in general parametric models (based on gefp) is available in sctest.default.

# Value

An object of class "htest" containing:

```
statistic the test statistic,
```

p.value the corresponding p value,

method a character string with the method used, data.name a character string with the data name.

#### See Also

```
sctest.efp, sctest.Fstats, sctest.default
```

```
## Example 7.4 from Greene (1993), "Econometric Analysis"
## Chow test on Longley data
data("longley")
sctest(Employed ~ Year + GNP.deflator + GNP + Armed.Forces, data = longley,
    type = "Chow", point = 7)

## which is equivalent to segmenting the regression via
fac <- factor(c(rep(1, 7), rep(2, 9)))
fm0 <- lm(Employed ~ Year + GNP.deflator + GNP + Armed.Forces, data = longley)</pre>
```

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```
fm1 <- lm(Employed ~ fac/(Year + GNP.deflator + GNP + Armed.Forces), data = longley)
anova(fm0, fm1)
## estimates from Table 7.5 in Greene (1993)
summary(fm0)
summary(fm1)</pre>
```

sctest.Fstats

supF-, aveF- and expF-Test

# Description

Performs the supF-, aveF- or expF-test

# Usage

```
## S3 method for class 'Fstats'
sctest(x, type = c("supF", "aveF", "expF"),
    asymptotic = FALSE, ...)
```

# **Arguments**

x an object of class "Fstats".

type a character string specifying which test will be performed.

asymptotic logical. Only necessary if x contains just a single F statistic and type is "supF"

or "aveF". If then set to TRUE the asymptotic (chi-square) distribution instead

of the exact (F) distribution will be used to compute the p value.

... currently not used.

#### **Details**

If x contains just a single F statistic and type is "supF" or "aveF" the Chow test will be performed.

The original GAUSS code for computing the p values of the supF-, aveF- and expF-test was written by Bruce Hansen and is available from https://www.ssc.wisc.edu/~bhansen/. R port by Achim Zeileis.

#### Value

An object of class "htest" containing:

statistic the test statistic,

p. value the corresponding p value,

method a character string with the method used, data.name a character string with the data name.

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# References

Andrews D.W.K. (1993), Tests for parameter instability and structural change with unknown change point, *Econometrica*, **61**, 821-856.

Andrews D.W.K., Ploberger W. (1994), Optimal tests when a nuisance parameter is present only under the alternative, *Econometrica*, **62**, 1383-1414.

Hansen B. (1992), Tests for parameter instability in regressions with I(1) processes, *Journal of Business & Economic Statistics*, **10**, 321-335.

Hansen B. (1997), Approximate asymptotic p values for structural-change tests, *Journal of Business & Economic Statistics*, **15**, 60-67.

#### See Also

```
Fstats, plot. Fstats
```

#### **Examples**

```
## Load dataset "nhtemp" with average yearly temperatures in New Haven
data(nhtemp)
## plot the data
plot(nhtemp)

## test the model null hypothesis that the average temperature remains
## constant over the years for potential break points between 1941

## (corresponds to from = 0.5) and 1962 (corresponds to to = 0.85)

## compute F statistics
fs <- Fstats(nhtemp ~ 1, from = 0.5, to = 0.85)

## plot the F statistics
plot(fs, alpha = 0.01)

## and the corresponding p values
plot(fs, pval = TRUE, alpha = 0.01)

## perform the aveF test
sctest(fs, type = "aveF")</pre>
```

solveCrossprod

Inversion of X'X

# Description

Computes the inverse of the cross-product of a matrix X.

#### Usage

```
solveCrossprod(X, method = c("qr", "chol", "solve"))
```

# Arguments

X a matrix, typically a regressor matrix.

method a string indicating whether the QR decomposition, the Cholesky decomposition

or solve should be used.

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# **Details**

Using the Cholesky decomposition of X'X (as computed by crossprod(X)) is computationally faster and preferred to solve(crossprod(X)). Using the QR decomposition of X is slower but should be more accurate.

# Value

a matrix containing the inverse of crossprod(X).

# **Examples**

```
X <- cbind(1, rnorm(100))
solveCrossprod(X)
solve(crossprod(X))</pre>
```

SP2001

S&P 500 Stock Prices

# Description

A multivariate series of all S&P 500 stock prices in the second half of the year 2001, i.e., before and after the terrorist attacks of 2001-09-11.

# Usage

```
data("SP2001")
```

# **Format**

A multivariate daily "zoo" series with "Date" index from 2001-07-31 to 2001-12-31 (103 observations) of all 500 S&P stock prices.

# **Source**

Yahoo! Finance: https://finance.yahoo.com/.

# References

Zeileis A., Leisch F., Kleiber C., Hornik K. (2005), Monitoring Structural Change in Dynamic Econometric Models, *Journal of Applied Econometrics*, **20**, 99–121.

#### See Also

```
get.hist.quote
```

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```
## load and transform data
## (DAL: Delta Air Lines, LU: Lucent Technologies)
data("SP2001")
stock.prices <- SP2001[, c("DAL", "LU")]
stock.returns <- diff(log(stock.prices))</pre>
## price and return series
plot(stock.prices, ylab = c("Delta Air Lines", "Lucent Technologies"), main = "")
plot(stock.returns, ylab = c("Delta Air Lines", "Lucent Technologies"), main = "")
## monitoring of DAL series
myborder <- function(k) 1.939*k/28</pre>
x <- as.vector(stock.returns[, "DAL"][1:28])</pre>
dal.cusum <- mefp(x \sim 1, type = "OLS-CUSUM", border = myborder)
dal.mosum <- mefp(x \sim 1, type = "OLS-MOSUM", h = 0.5, period = 4)
x <- as.vector(stock.returns[, "DAL"])</pre>
dal.cusum <- monitor(dal.cusum)</pre>
dal.mosum <- monitor(dal.mosum)</pre>
## monitoring of LU series
x <- as.vector(stock.returns[, "LU"][1:28])</pre>
lu.cusum \leftarrow mefp(x \sim 1, type = "OLS-CUSUM", border = myborder)
lu.mosum \leftarrow mefp(x ~ 1, type = "OLS-MOSUM", h = 0.5, period = 4)
x <- as.vector(stock.returns[, "LU"])</pre>
lu.cusum <- monitor(lu.cusum)</pre>
lu.mosum <- monitor(lu.mosum)</pre>
## pretty plotting
## (needs some work because lm() does not keep "zoo" attributes)
cus.bound <- zoo(c(rep(NA, 27), myborder(28:102)), index(stock.returns))</pre>
mos.bound <- as.vector(boundary(dal.mosum))</pre>
mos.bound <- zoo(c(rep(NA, 27), mos.bound[1], mos.bound), index(stock.returns))</pre>
## Lucent Technologies: CUSUM test
plot(zoo(c(lu.cusum$efpprocess, lu.cusum$process), index(stock.prices)),
 ylim = c(-1, 1) * coredata(cus.bound)[102], xlab = "Time", ylab = "empirical fluctuation process")
abline(0, 0)
abline(v = as.Date("2001-09-10"), lty = 2)
lines(cus.bound, col = 2)
lines(-cus.bound, col = 2)
## Lucent Technologies: MOSUM test
plot(zoo(c(lu.mosum$efpprocess, lu.mosum$process), index(stock.prices)[-(1:14)]),
 ylim = c(-1, 1) * coredata(mos.bound)[102], xlab = "Time", ylab = "empirical fluctuation process")
abline(0, 0)
abline(v = as.Date("2001-09-10"), lty = 2)
lines(mos.bound, col = 2)
lines(-mos.bound, col = 2)
## Delta Air Lines: CUSUM test
plot(zoo(c(dal.cusum$efpprocess, dal.cusum$process), index(stock.prices)),
```

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```
ylim = c(-1, 1) * coredata(cus.bound)[102], xlab = "Time", ylab = "empirical fluctuation process")
abline(0, 0)
abline(v = as.Date("2001-09-10"), lty = 2)
lines(cus.bound, col = 2)
lines(-cus.bound, col = 2)

## Delta Air Lines: MOSUM test
plot(zoo(c(dal.mosum$efpprocess, dal.mosum$process), index(stock.prices)[-(1:14)]),
    ylim = range(dal.mosum$process), xlab = "Time", ylab = "empirical fluctuation process")
abline(0, 0)
abline(v = as.Date("2001-09-10"), lty = 2)
lines(mos.bound, col = 2)
lines(-mos.bound, col = 2)
```

supLM

Generators for efpFunctionals along Continuous Variables

# **Description**

Generators for efpFunctional objects suitable for aggregating empirical fluctuation processes to test statistics along continuous variables (i.e., along time in time series applications).

# Usage

```
supLM(from = 0.15, to = NULL)
maxMOSUM(width = 0.15)
```

# **Arguments**

from, to

numeric from interval (0, 1) specifying start and end of trimmed sample period.

By default, to is 1 - from, i.e., with the default from = 0.15 the first and last 15 percent of observations are trimmed.

width

a numeric from interval (0,1) specifying the bandwidth. Determines the size of the moving data window relative to sample size.

#### **Details**

supLM and maxMOSUM generate efpFunctional objects for Andrews' supLM test and a (maximum) MOSUM test, respectively, with the specified optional parameters (from and to, and width, respectively). The resulting objects can be used in combination with empirical fluctuation processes of class gefp for significance testing and visualization. The corresponding statistics are useful for carrying out structural change tests along a continuous variable (i.e., along time in time series applications). Further typical efpFunctionals for this setting are the double-maximum functional maxBB and the Cramer-von Mises functional meanL2BB.

# Value

An object of class efpFunctional.

# References

Merkle E.C., Zeileis A. (2013), Tests of Measurement Invariance without Subgroups: A Generalization of Classical Methods. *Psychometrika*, **78**(1), 59–82. doi:10.1007/S11336-012-9302-4

Zeileis A. (2005), A Unified Approach to Structural Change Tests Based on ML Scores, F Statistics, and OLS Residuals. *Econometric Reviews*, **24**, 445–466. doi:10.1080/07474930500406053.

Zeileis A. (2006), Implementing a Class of Structural Change Tests: An Econometric Computing Approach. *Computational Statistics & Data Analysis*, **50**, 2987–3008. doi:10.1016/j.csda.2005.07.001.

Zeileis A., Hornik K. (2007), Generalized M-Fluctuation Tests for Parameter Instability, *Statistica Neerlandica*, **61**, 488–508. doi:10.1111/j.1467-9574.2007.00371.x.

## See Also

```
efpFunctional, gefp
```

# **Examples**

```
## seatbelt data
data("UKDriverDeaths")
seatbelt <- log10(UKDriverDeaths)</pre>
seatbelt <- cbind(seatbelt, lag(seatbelt, k = -1), lag(seatbelt, k = -12))
colnames(seatbelt) <- c("y", "ylag1", "ylag12")</pre>
seatbelt <- window(seatbelt, start = c(1970, 1), end = c(1984,12))
## empirical fluctuation process
scus.seat <- gefp(y ~ ylag1 + ylag12, data = seatbelt)</pre>
## supLM test
plot(scus.seat, functional = supLM(0.1))
## MOSUM test
plot(scus.seat, functional = maxMOSUM(0.25))
## double maximum test
plot(scus.seat)
## range test
plot(scus.seat, functional = rangeBB)
## Cramer-von Mises statistic (Nyblom-Hansen test)
plot(scus.seat, functional = meanL2BB)
```

USIncExp

Income and Expenditures in the US

#### **Description**

Personal income and personal consumption expenditures in the US between January 1959 and February 2001 (seasonally adjusted at annual rates).

# Usage

```
data("USIncExp")
```

#### **Format**

A multivariate monthly time series from 1959(1) to 2001(2) with variables income monthly personal income (in billion US dollars), expenditure monthly personal consumption expenditures (in billion US Dollars).

#### Source

```
https://web.archive.org/web/20201205041942/http://www.economagic.com/
```

#### References

A. Zeileis, F. Leisch, K. Hornik, C. Kleiber (2002), strucchange: An R Package for Testing for Structural Change in Linear Regression Models. *Journal of Statistical Software* 7(2), 1–38.

```
## These example are presented in the vignette distributed with this
## package, the code was generated by Stangle("strucchange-intro.Rnw")
### chunk number 1: data
data("USIncExp")
plot(USIncExp, plot.type = "single", col = 1:2, ylab = "billion US$")
legend(1960, max(USIncExp), c("income", "expenditures"),
    lty = c(1,1), col = 1:2, bty = "n")
### chunk number 2: subset
data("USIncExp")
USIncExp2 <- window(USIncExp, start = c(1985,12))
### chunk number 3: ecm-setup
coint.res <- residuals(lm(expenditure ~ income, data = USIncExp2))</pre>
coint.res \leftarrow lag(ts(coint.res, start = c(1985,12), freq = 12), k = -1)
USIncExp2 <- cbind(USIncExp2, diff(USIncExp2), coint.res)</pre>
USIncExp2 <- window(USIncExp2, start = c(1986,1), end = c(2001,2))
colnames(USIncExp2) <- c("income", "expenditure", "diff.income",</pre>
                  "diff.expenditure", "coint.res")
ecm.model <- diff.expenditure ~ coint.res + diff.income
### chunk number 4: ts-used
plot(USIncExp2[,3:5], main = "")
```

```
### chunk number 5: efp
ocus <- efp(ecm.model, type="OLS-CUSUM", data=USIncExp2)</pre>
me <- efp(ecm.model, type="ME", data=USIncExp2, h=0.2)</pre>
### chunk number 6: efp-boundary
bound.ocus <- boundary(ocus, alpha=0.05)</pre>
### chunk number 7: OLS-CUSUM
plot(ocus)
### chunk number 8: efp-boundary2
plot(ocus, boundary = FALSE)
lines(bound.ocus, col = 4)
lines(-bound.ocus, col = 4)
### chunk number 9: ME-null
plot(me, functional = NULL)
### chunk number 10: efp-sctest
sctest(ocus)
### chunk number 11: efp-sctest2
sctest(ecm.model, type="OLS-CUSUM", data=USIncExp2)
### chunk number 12: Fstats
fs <- Fstats(ecm.model, from = c(1990, 1), to = c(1999,6), data = USIncExp2)
```

```
### chunk number 13: Fstats-plot
plot(fs)
### chunk number 14: pval-plot
plot(fs, pval=TRUE)
### chunk number 15: aveF-plot
plot(fs, aveF=TRUE)
### chunk number 16: Fstats-sctest
sctest(fs, type="expF")
### chunk number 17: Fstats-sctest2
sctest(ecm.model, type = "expF", from = 49, to = 162, data = USIncExp2)
### chunk number 18: mefp
USIncExp3 <- window(USIncExp2, start = c(1986, 1), end = c(1989, 12))
me.mefp <- mefp(ecm.model, type = "ME", data = USIncExp3, alpha = 0.05)</pre>
### chunk number 19: monitor1
USIncExp3 <- window(USIncExp2, start = c(1986, 1), end = c(1990, 12))
me.mefp <- monitor(me.mefp)</pre>
### chunk number 20: monitor2
USIncExp3 <- window(USIncExp2, start = c(1986, 1))
me.mefp <- monitor(me.mefp)</pre>
me.mefp
```

```
### chunk number 21: monitor-plot
plot(me.mefp)
### chunk number 22: mefp2
USIncExp3 <- window(USIncExp2, start = c(1986, 1), end = c(1989, 12))
me.efp <- efp(ecm.model, type = "ME", data = USIncExp3, h = 0.5)
me.mefp <- mefp(me.efp, alpha=0.05)</pre>
### chunk number 23: monitor3
USIncExp3 <- window(USIncExp2, start = c(1986, 1))</pre>
me.mefp <- monitor(me.mefp)</pre>
### chunk number 24: monitor-plot2
plot(me.mefp)
```

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