

Weighted Support Vector Machine Formulation

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October 11, 2024

The original formulation of unweighted SVM with linear kernel is as follows Valdimir and Vapnik (1995):

$$\begin{aligned}
 \min_{\omega, \xi} \quad & \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^n (\xi_i + \xi_i^*) \\
 \text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i, \\
 & \langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*, \\
 & \xi_i, \xi_i^* \geq 0.
 \end{aligned}$$

The constant $C > 0$ determines the trade-off between the flatness of f and the amount up to which deviations larger than ε are tolerated. This corresponds to dealing with a so called ε -insensitive loss function $|\xi|_\varepsilon$ described by

$$|\xi|_\varepsilon = \begin{cases} 0, & \text{if } |\xi| \leq \varepsilon \\ |\xi| - \varepsilon, & \text{o/w.} \end{cases}$$

The corresponding weighted SVM with W_i as individual weights:

$$\begin{aligned}
 \min_{\omega, \xi} \quad & \frac{1}{2} \|\omega\|^2 + C \sum_{i=1}^n W_i (\xi_i + \xi_i^*) \\
 \text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i, \\
 & \langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*, \\
 & \xi_i, \xi_i^* \geq 0.
 \end{aligned}$$

Other kinds of weighted SVMs (with different kernels) have the similar formulation.

Available kernels:

kernel	formula	parameters
linear	$\mathbf{u}^\top \mathbf{v}$	(none)
polynomial	$(\gamma \mathbf{u}^\top \mathbf{v} + c_0)^d$	γ, d, c_0
radial basis fct.	$\exp\{-\gamma \mathbf{u} - \mathbf{v} ^2\}$	γ
sigmoid	$\tanh\{\gamma \mathbf{u}^\top \mathbf{v} + c_0\}$	γ, c_0

References

V Valdimir and N Vapnik. The nature of statistical learning theory. 1995.